Behavioral finance perspective of portfolio theory, where do we stand?

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1. INTRODUCTION

One of the main contributions of Behavioral Finance to financial economics (if not the most important) has been the introduction of empirical findings from psychology that catalogue some of the ways in which economic agents form expectations and make decisions. With an extensive categorization of behavior biases and with prospect theory as the alternative approach to utility theory, financial economist have been able to formalize models of bounded rationality that do a better job than those proposed by the traditional theory in explaining different phenomena present in the financial markets.

Over the last decade Behavioral Finance has formalized several arguments related to portfolio theory in which the framework of selection and allocation is structured around models that evaluate the impact of particular outcome values in choice -a necessary adjustment under the assumption of limited rationality. Although common elements and ideas can be traced within the existing literature, to date –as in many other topics studied by this young field of finance- there is not a formal and homogeneous body of theory reflecting the contributions made so far. Thus, the main purpose of this work is to outline the common ideas present in the literature and to show that by implementing alternative approaches to expected utility theory, Behavioral portfolio models tend to evolve towards an N-Fund version of the allocation process. The paper analyzes some of the most recent ideas of Behavioral Finance on portfolio theory and compares them with some of the earlier contributions as well as with their counterparts in the classical approach in order to determine the current state of the research and its significance. Additionally, based on this thoughtful review, the paper will hypothesize about the future course of the research in this particular area of financial theory.

The fundamental motivation for a behavioral approach to portfolio theory stems from different sources; in first place it is related to the proven deficiencies of Expected Utility (UE) theory as an adequate framework for modeling the behavior of economic agents. Indeed, the bounded rationality assumption of Behavioral approximations implies that the decision process of such individuals is not always consistent with the principles of EU[1] and that their updating process according to Bayes' Law is usually biased or inaccurate. As an alternative to this perspective Behavioral Finance based models have used Prospect Theory as well as

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[1] Order, transitivity, invariance, dominance, cancellation and continuity.

Security-Potential/Aspiration as models of risky choice under uncertainty.

In second place the multidimensional nature of risk and its relation to individual preferences and time have not been consistently embodied within classic portfolio theory. The typical approximation based on the specification of a mathematical relation representing the utility function that contains a factor measuring individual's risk profile (usually the "Arrow-Pratt" coefficient), is not necessarily a comprehensive specification towards the design of portfolios congruent with the changing preferences and probability beliefs in the short and medium terms. The limitations of this point of view worsen if one thinks that investor's decisions are frequently influenced by sentiments, goals and timeframes that are in permanent evolution.

An additional and very important motivation derives from the mean-variance optimization criterion distinctive of the classical approach. While from a mathematical and statistical point of view this is a straightforward idealization of the trade-off relation between return and a partial measure of risk, it is not the only way to visualize the problem of portfolio construction as it was early recognized by Roy (1952) and Markowitz (1959)[2], who considered the possibility of solving the portfolio construction puzzle based on a defensive scheme that may be not mathematically equivalent to the conventional optimization method. More recently, Oden & Lopes (1989,1999) and Oden (1994) study the role of sentiments in risky choices shedding some light on alternative approximations to the conceptualization of the risk-return relation.

The background of the idea of a behavioral model applied to portfolio selection using an alternative model of choice under uncertainty was first outlined by Bernartzi and Thaler (1995) who studied the implication of loss aversion on portfolio choice and the equity premium. They consider a one-period model with multiple simulations in which an investor with prospect theory type preferences allocates an optimal mix of stocks and T-Bills. As it will be explained in deeper detail below, Prospect theory argues that when choosing between gambles people compute the gains and losses for each one and select the one with the highest prospective Utility. The authors conclude that myopic loss aversion drives the allocation and make investors unwilling to allocate a higher proportion of their wealth on stocks even in presence of an equity premium in the medium and long terms.

Over the last decade the amount and quality of the research done on behavior-

[2] The theory proposed by Roy has become better known as "Safety First Theory", while the alternative algorithm proposed by Markowitz is also known as semi-variance optimization.

al models of portfolio theory has grown significantly. Some of the key elements that can be identified as common ideas within the literature on the topic can be summarized as follows:

- Most of the models use one or more of the alternative theories of choice under uncertainty, plus a set of behavior biases in order to define a conceptual framework that guides the process of portfolio construction.
- Usually the risk preferences are described in terms of investor's beliefs and economic goals, this approach is congruent with a multidimensional conception of risk profiles and allows a different conception of the portfolio problem.
- Based on these notions, the models assume investors that use points of reference and levels of wealth in their interpretation of the portfolio selection and allocation tasks. As it will be shown this idea is congruent with the mental accounting concept first documented by Thaler (1990).

It is to be said that although most of the contributions of Behavioral finance to portfolio theory are in the form of analysis of portfolio decision at the individual level, there is however an increasing number of normative approaches that eventually could become the basis of a formal Behavioral portfolio theory extensible to different agents (i.e. Institutional Investors).

The paper is organized as follows. The first section presents the theoretical framework of prospect theory and Security-Potential/Aspiration theory as a representative sample of descriptive models of risky choice under uncertainty used as counterparts to EU. The second part introduces formal models and theories that attempt to formalize the behavioral solution to the portfolio construction problem, this section makes an special emphasis on the mathematical aspect of one of the models. The third section summarizes some of the characteristics of the portfolios chosen by investors using the behavioral approach. Finally the fourth section concludes on the eventual course of the research and the concrete contributions of behavioral finance to portfolio theory.

2. DESCRIPTIVE MODELS OF RISKY CHOICE AND THEIR ROLE IN THE BEHAVIORAL FINANCE APPROACH TO PORTFOLIO THEORY

Like many other traditional economic and financial models, classical portfolio theory finds in Expected Utility a suitable foundation for decision making under uncertainty. In essence EU asserts that when people choose between alternative probability distributions over outcomes they should and do choices as to maximize a function of the following form:

$$E(U) = \sum p_i \, U(v_i)$$

In which *U(vi)* represent the numerical "utilities" associated with each one of the possible outcomes and the *pi* are the probabilities associated with them. As Markowitz (1959) points out, this idea implies that the individual should:

- **a.** Acts on the basis of expected utility
- **b.** Never makes an error in logic or arithmetic; and
- **c.** Inevitably chooses a portfolio which is efficient in terms of an analysis based on mean and variance;

The rational behavior of economic agents assumed under this vision is common to most of the economic models developed under the precepts of the Theory of games of Von Newman –Morgenstern(1944) that at the same time are an extension of Bernoulli's(1738) notion of marginal decreasing utility. As Markowitz(1991) indicates: "although there is no inevitably connection between the validity of the expected utility maxim and the validity of portfolio analyses based on, say, expected return and variance(…)the inspection of the utility curve associated with a measure of risk can direct us to peculiarities in its evaluation of portfolios"[3]. And therefore, it has been used as the formal approximation in the classical theory.

For decades economists ignored the weaknesses of this representation assuming that the deviations from rational behavior were due more to subject failures than to inconsistencies of the model itself. Nevertheless, the work of several psychologist and economists such as Allais's(1953), Camerer(1989), and D. Kahneman, A. Tversky (1979, 1992), among others have unveiled major phenomena of choice which violate the standard predictions of EU and that can be summarized as follows:

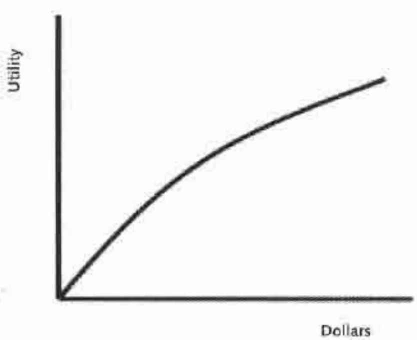
- Framing Effects: Expected Utility assumes description invariance (equivalent formulations of choice problem should give rise to the same preference order). However there is much evidence that variations in the framing of alternatives yield systematically different preferences.
- Nonlinear Preferences: The Expectation principle asserts that the utility of a risky prospect is linear in outcome probabilities. Contrary to this assumption it has been demonstrated that the difference between probabilities of, say, 0.99 and 1 has more impact on preferences than the difference between 0.10 and 0.11.
- Source Dependence: People's willingness to bet on an uncertain event depends not only on the degree of uncertainty but also on its source. People prefer to bet on an urn containing equal numbers of red and green balls,

[3] Markowitz H. "Portfolio Selection: Efficient Diversification of Investments", NY. 1991.

rather than on an urn that contains red and green balls in unknown proportions.

- Risk Seeking: Although risk aversion has been the general assumption when it comes to modeling the behavior of economic agents, risk seeking conducts are prevalent when people must choose between a sure loss and a substantial probability of a larger loss.
- Loss aversion: One of the basic phenomena of choice under uncertainty or risk losses loom larger than gains, therefore people tend to be loss averse.

Figure1. Bernoulli´s Utility Function



From the violations of the principles of EU listed above, non-linearity of preferences is one of the most troublesome. In fact, the difference between the impacts of a change in probabilities at distinct levels motivated the work on nonlinear modifications of the UE approach. These theories have been grouped under the denomination of "Decumulatively Weighted Utility Approaches" with Prospect Theory and Security-Potential/Aspiration standing as the two most frequent references in the Behavioral Finance literature on alternative formulations of portfolio theory.[4]

The idea of decumulative weighted utility is straightforward, it transforms raw probabilities to decumulative probabilities:

$$DWU = \sum h(\sum p_j)\ (U(v_j) - U(v_{j-1}))$$

$$= \sum h(D_j)\ (U(v_j) - U(v_{j-1}))$$

The *vi* (possible outcomes) are ordered from lowest to highest. $Di = \sum pi$ is the decumulative probability associated with the outcome *vi*, that is *Di* is the probability of obtaining an outcome at least as high as outcome *vi*. For instance D_1 (the decumulative probability of the worst outcome v1) is 1 (the individual gets at least that for sure) and D_{n+1} (the decumulative probability of exceeding the best outcome, *vn*) is zero. The function *h* maps decumulative probabilities on to a range (0,1) and so preserves dominance. It also

[4] For a complete treatment of Decumulative Weighted Utility Approaches and a comparative analysis of utility theories see: Lopes L. " The role of Aspiration Level in Risky Choice: A Comparison of cumulative prospect Theory and SP/A Theory".

provides a parameter for modeling the curvature of the utility function and therefore the attitudes towards risk, namely, aversion, neutrality and proclivity.

Prospect theory as a decumulative weighted utility model

The predictions and concepts related to Prospect Theory derive from direct empirical evidence about violations to the EU theory. At the very bone of the discussion Prospect theory argues that individuals decide over reference points where gains or losses can occur rather than over final value of assets. This assertion contradicts Bernoulli's Idea of Marginal expected utility in which individual were believed to care about the pre-existing level of wealth. As Kahneman and Tversky point out, the key elements of Prospect Theory are: "1) a value function that is concave for gains, convex for losses and steeper for losses than for gains, and 2) a nonlinear transformation of the probability scale, which overweights small probabilities and underweights moderate and high probabilities."[5]

Kahneman and Tversky (1992) reformulate the initial version of Prospect Theory, using decumulative weighted utility in order to consider gambles and economic problems with two or more outcomes. The theory proposes that people assign values to risky prospects by means of the following representation:

$$\sum v(x_i) \, (U(v_i) - U(v_{i+1}))$$

In this version the Utility function remains unchanged, being concave (risk averse) for gains and convex(risk seeking) for losses, with the loss function assumed to be steeper than the gain function (λ–1):

$$v = \begin{matrix} x^{\alpha} & \text{if} & x \geqslant 0 \\ -\lambda(-x)^{\alpha} & \text{if} & x < 0 \end{matrix}$$

Note that λ is the coefficient of loss aversion, and α is the exponent of the value function v which magnifies the impact of the outcomes. The decumulative Weighting functions take a functional form that allows them to encompass both concave and convex regions accordingly to the assumed behavior towards risk. For gains the function reinforces risk aversion for most of the gambles and choices faced, however it tends towards risk seeking behavior for gambles that have small probabilities of large outcomes:

$$w(P) = \frac{P^{\gamma}}{(P^{\gamma} + (1-P)^{\gamma})^{1/\gamma}}.$$

For losses the weighting function is cumulative (therefore the name of the article: "Cumulative Representation of Uncertainty", so that it reinforces risk seeking

[5] D. Kahneman, A. Tversky (1992). "Advances in Prospect Theory: Cumulative Representation of uncertainty".

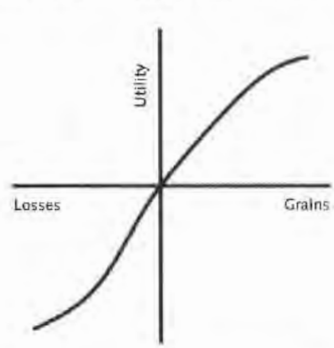
for most of the gambles but tends to risk aversion for lottery-type alternatives. Through empirical evidence the authors estimate the values for the parameters involved in the model as indicated below:

Tabla 1

α	0.88
λ	2.25
γ	0.61
δ	0.69

The high value of the term λ indicates the importance of the loss aversion hypothesis and also validates the finding that the impact of a loss is twice that of a gain, which in psychological terms implies that a loss of a certain amount of money will only be mentally recovered by a gain that is twice the value of such shortfall. Kahneman and Tversky contributions can be summarized by indicating that economic agents are actually affected by emotion which often destroys the self-control -an essential tool for rational decision making. Moreover, in the aftermath of their work it has become clearer that people are often unable to fully understand the type of dynamic trade-offs and decisions that they are dealing with. They experience what psychologists call cognitive difficulties.

Figure 2: Prospect Theory Utility Function



Security-Potential/Aspiration as a decumulative weighted utility model

Lopes(1987-1990) introduced SP/A as a descriptive theory with a dual choice criterion in which the process of choosing between gambles conduces to the analysis of the trade-off relationship between two values: Security and Potential, considered both within a certain aspiration level (a concept analogous to the reference point(s) in Prospect Theory). Mathematically this could be represented as:

$$U(E_h(W), D(A))$$

Security and Potential are included because each seems necessary to adequately capture human choices under risk. The double criterion is modeled by the following decumulative weighted value rule:

$$E_h(W)= SP =\sum h(D_j)\ (\ U(\ v_j) - U(v_{j-1}))$$

Where the decumulative weighting

function, $h(Di)$ has the form:

$$h(Di) = wDsq+1 + (1-w)\,[1-(1-Dp)q+1]$$

for both gains and losses. The equation is derived from the idea that individuals assign values to the gambles based on 2 sentiments: fear and hope which are underlying factors behind the Security and the Potential incentives. The Utility derived from choices is captured through the exponential terms, q, associated with the probability D_s and D_p respectively. In this way if fear is the driven factor, disproportionate weight is assigned to higher values of D_s (the ones that attach to lower outcomes in the gamble), conversely, if hope is the dominant factor, a disproportionate weight is assigned to the lower values of D_p (the ones that attach to higher outcomes). The parameter w determines the relative weight of the S and P analyses. If $w=1$, the decision maker is strictly security-minded, if $w=0$ the decision maker is potentially minded. If $0<w>1$ the decision maker is cautiously minded.

Up to this point the analysis of the risky outcome completes its first stage, the second one involves the term $D(A)$, the probability that the payoff will be A or higher. This aspiration criterion operates on the principle of stochastic control in which subjects are assumed to asses the attractiveness of gambles by the probability that it will yield an outcome at or above the aspiration level, therefore a risky prospect is more attractive if it has a lower probability of falling below a certain aspiration level. In this sense SP/A theory is very similar to the safety first Theory of Roy (1952) in which the utility derives from reaching returns above a certain subsistence level.

A careful review of the model described in this section indicates that they both model the process of decision making through a decumulative weighting rule, and they both include a point on the value dimension that has special significance to the choice maker, in Prospect theory this point is zero, while in SP/A theory it is a certain aspiration level. The later model however is the only one to account for stochastic dominance.

With the background provided by these theories one could ask what type of motivation would force the inclusion of new paradigms of choice under uncertainty within Portfolio Theory? A basic one is that risk perception has a high impact on beliefs and preferences and therefore the framework used to model such perceptions has to be a very realistic one. As it has been shown measurement of the impact of risk and uncertainty becomes complicated when the parameters are fluid rather than stationary. A single measure of risk aversion such as the "Arrow Pratt" coefficient does not provide enough information about risk appetite. More over risk is a multidimensional variable that evolves through time; few people feel the same about risk every day of their lives. As individuals grow older, richer, or poorer, their perception of what risk is as well as their level of aversion to it will shift. Investors as a group also alter their views about risk, causing significant changes in how they value the future streams of earnings that they expect stocks and long term bonds to provide, therefore a more realistic theory about choice is more than justified in the context of an alternative approximation to portfolio theory.

3. FORMAL MODELS OF PORTFOLIO SELECTION: THE BEHAVIORAL FINANCE APPROACH

It has become a methodological characteristic within Behavioral finance to study the financial phenomena recurring to conceptualizations related to either the formation of Beliefs, the configuration of preferences and its effect on choice or in some cases a mixture of both. According to this, a common feature of the models referenced below is their use of Prospect Theory or SP/A Theory as the conceptual tools for the study of individual preferences and choice, in parallel with the use of a particular subset of systematic biases that arise when people form their beliefs[6].

Two additional concepts referenced within the models described are mental accounting and narrow framing. The first one refers to the process by which people assign separate mental accounts to different types of cash flows, for instance a popular mental category distinguishes between gains and losses. In the preceding section it was shown how these categories are mentally differentiated and perceived according to recent experiences, therefore they are referred as separate mental accounts.

On the other hand, Narrow framing refers to the tendency to treat individual gambles separately from other portions of wealth, people do not evaluate risky choices in addition to previous decisions, and on the contrary they tend to value each choice at a time without considering the effect of that particular decision added to the previous ones. An important consideration is that mental accounting and narrow framing are consistent with the idea of an N-fund separation theorem (the counter part to the 2 fund separation theorem of classical portfolio theory) and this fact is used in one way or another by the models presented below.

a. The Behavioral Portfolio Theory of Shefrin And Statman

Behavioral Portfolio Theory (BPT), introduced by Shefrin and Statman (1999), is the first positive portfolio theory formulated under a Behavioral perspective. In this theory the two decumulative

[6] For a complete classification of these biases see: Barberis, N. Thaler, R. (2003), Shefrin(2000).

approximations to choice under uncertainty are combined with the principle of mental accounting in order to formulate a portfolio selection model under the assumption of limited rationality. The authors make use of SP/A theory as the general model of choice but incorporate the use of reference points and the process of mental accounting from Prospect Theory.

The basic idea is that a behavioral approximation to portfolio selection and allocation should be observant of the role played by fear and hope as emotions that control the Security and Potential trade off. At the same time the Aspiration level (*A*) imposes a restriction to the conformation of the portfolio. Therefore, while mean-variance investors face a substitutive relation between returns (m) and standard deviation (s) investors in BPT try to balance the trade off between *Eh(W)* and *Pr[W£A]*. The utility function takes the same specification form as in Lopes (1994):

$$U(E_h(W), D(A)),$$

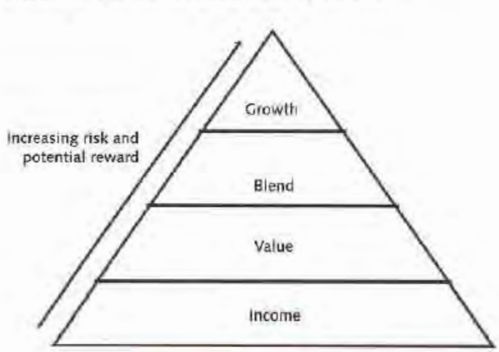
$$h(D_i) = wD_s^{qs+1} + (1-w)\,[1-(1-D_p)^{qp+1}]$$

One of the main advantages of the theory in terms of innovation is that risk is interpreted as a multidimensional variable dependent on five parameters: first *qs*, which measures the strength of fear (need of security), *qp* which measures the strength of hope (need for potential), *A*, the aspiration level, *w*, reflects the strength of fear relative to hope and the term ≥ which determines the strength of fear relative to hope.

The authors consider two versions of BTP, one with a single mental account with 2 layers, a downside protection, and an upside potential layer. In this adaptation investors consider the covariance between accounts. In the second version, Investors have multiple mental accounts and therefore they divide their money into many layers corresponding to many different goals and levels of aspiration. In this later adaptation investors act as if they overlooked covariances. The basic idea here stems from Kahneman and Tversky's hypothesis according to which due to the difficulty that covariance and other properties of joint probability distributions impose in mental processes people tend to simplify the math by recurring to the use mental accounts and heuristic biases (i.e rules of thumb).

The most basic version of BPT asserts that the Dubind-Savage type investors (investors who buy insurance policies while they also buy lottery tickets) are not risk seekers, instead they are investors with high aspiration levels that are willing to invest in securities with high but unstable pay-offs. Thus, accordingly to behavioral portfolio theory, investors view their portfolios not as whole, as prescribed by mean-variance portfolio theory, but as distinct layers in a pyramid of assets, each layer is associated with the particular goals of Security and Potential as well as with an attitude towards risk. One layer might be a "downside protection" layer, designed to protect investors from being poor. Another might be an "upside potential" layer, designed to give investors a chance at being rich.

Figure 3: Portfolios as Layered Pyramids



Because of this structure the concept of efficient frontier in BPT is different from that on Mean-variance theory. Indeed, and as pointed out before, Behavioral investors construct the behavioral efficient frontier by identifying the portfolios with the highest level of expected wealth for each probability that wealth would fall below the aspiration level. Consequently the Standard deviation is replaced by a probability measure of falling below the aspiration level as the main risk measure.

In a recent paper, Statman (2003) provides evidence about Asset management companies that actually recommend investors to build their portfolio in a very similar fashion to the one of BPT. The strategies documented include the investment pyramid that the Putnam mutual fund company (2003) prescribes to its investors. This structure is also reflected by the "core and satellite" strategy offered by Charles Schwab and the "risk budget" methodology used by other well known Investment Banks.

b. Loss Aversion and the demand For Risky Assets

Unlike Behavioral Portfolio Theory this model studies optimal portfolio allocation behavior of a loss-averse investor. According to Gomes (2000) the demand for Risky assets as well as the behavior depend crucially on the level of surplus wealth (current wealth relative to the reference point), and on how the investor's reference point reacts to changes in the current price of the assets. As surplus wealth reaches a certain threshold, the investor sells a significant part of his stock holdings and follows a (generalized) portfolio insurance rule, protecting himself against losses (relative to his reference point). Intuitively, as the stock price goes up, the investor faces a trade-off between the potential benefit from insuring himself against losses and the cost of doing so: selling a large share of his portfolio and giving up the equity premium.

As the stock price rises further, and surplus wealth keeps increasing, the cost of switch to the portfolio insurance rule becomes smaller, consequently, the investor doesn't have to sell as many stocks. Therefore, as the price raises enough the agent eventually switches to these type of assets. This generates a behavior consistent with the disposition effect: investors have a larger tendency to sell their winners and to hold on to their losers. In addition, this provides a rational motivation for portfolio insurance strategies, and identifies the conditions under which investors are more or less likely to follow these strategies.

According to Gomes a limitation of the typical specification of the value function in Kahneman an Tversky's original version of Prospect theory is that it implies that marginal utility is decreasing as wealth approaches zero or in other words that when facing losses beyond a certain limit the investor's utility function will become convex just as in the domain of gains. Consequently the proposed utility function takes the following form:

$$V \equiv \begin{cases} V_G \equiv \frac{(W-\Gamma))^{1-\gamma}}{1-\gamma} & W \geq \Gamma \\ \lambda V_L \equiv -\lambda\frac{(\Gamma-W)^{1-\gamma}}{1-\gamma} & \underline{W} < W < \Gamma \\ V_{BL} \equiv \frac{W^{1-\rho}}{1-\rho} - \left(\lambda\frac{(\Gamma-\underline{W})^{1-\gamma}}{1-\gamma} + \frac{\underline{W}^{1-\rho}}{1-\rho}\right) & W \leqslant \underline{W} \end{cases}$$

Where $\underline{W}$ identifies the level of wealth beyond which the utility function becomes concave, Γ is the point of reference and the other parameters are the same as those in the original version of prospect theory. This extended formulation allows for the fact that, for big enough losses ($W < \underline{W}$), decreasing marginal utility (of consumption) eventually dominates the psychological effect of the loss. This puts a limit on the amount of risk that the investor is willing to take, whenever he is in a losing position.

At this point it is convenient to mention that, as the models presented indicate, one of the basic criticisms that can be done to the original formulation of prospective theory (and even to the cumulative version of it) is that it does not imposes a limit to the loss adversity. Hence, most of the models that use prospective theory eventually conclude that for big losses the investor becomes risk averse instead of risk seeker.

The basic conclusions of this approach are as follows: 1)Loss-averse investors will abstain from holding equities unless they expect the equity premium to be quite large. 2)Loss-averse investors can generate a significant degree of trading volume even if they have homogeneous preferences and even if they are a small fraction of the population of investors. 3) When the loss-averse investors are following the generalized portfolio insurance strategy, then trading volume is positively correlated with stock return volatility. Intuitively, when the demand for portfolio insurance is stronger, the aggregate demand for stocks becomes more elastic therefore increasing both the volatility of returns and trading volume.

c . Portfolio Choice under loss aversion

This model presents a formal solution towards the portfolios of loss averse investors assuming a complete market and a general Ito process for the dynamic of the asset prices. Again Prospect Theory is used as the model of choice under uncertainty. A basic finding by Berkeleaar and Kouenberg (2003) is that first order risk aversion introduced by loss aversion reduces the proportion of the portfolio in stocks. Additionally through an empirical experiment the authors find that the 2.25 loss aversion parameter is close to the implied parameter using real data.

The model starts by defining the dynamics of the riskless asset (zero-th asset) by:

$dSo(t) = r(t)So(t)dt,$

where *So(t)*= Price of the asset at time *zero, r(t)*=risk free rate at time t and *dt* = time interval between time zero and time *n* and time *n +1*

The remaining assets are assumed to follow a classical Brownian motion given by:

$dSk(t)=\mu k(t)Sk(t)dt + \sigma\surd k(t)Sk(t)dB(t),$
$k = 1, . . . ,K,$

where μk= mean return on kth asset; σk= Volatility of the kth asset; *B(t)*= a standard wiener process.

The dynamics of the portfolio are then described by the following expression:

$$dW(t) = r(t)W(t)dt + (\mu(t) -\_r(t))ow(t)W(t)dt + \_(t)ow(t)W(t)dB(t)$$

It follows then that the investor attempts to maximize the expected utility of its wealth , W(T) at time horizon T using the expression:

$$Max\ E[U(W(T))]$$
$$s.t.$$
$$dW(t) = r(t)W(t)dt + (\mu(t) - ir(t))'w(t)W(t)dt + s(t)'w(t)W(t)dB(t)$$
$$W(t) \geq 0,\ 8\ t\ 2\ [0, T]$$

Using this framework the authors derive closed end form solutions for the optimal asset allocation conditions under loss aversion using the same utility function of Kahneman and Tversky(1992):

$$U(x) = \begin{cases} -2.25(-x)^{0.88} & x < 0 \\ x^{0.88} & x > 0 \end{cases}$$

Based on the mathematical solution to the proposed model the authors conclude that[7]:

- The optimal strategy for a loss averse investor can be decompose in a probability maximizing strategy and a growth strategyY. A loss averse investor aims at

[7] For a brief explanation of the mathematical argument please refer to appendix 1Y note the similitude between this conclusion and the one deduced by Behavioral portfolio theory.

maximizing the probability of reaching his aspiration level and desires some upside by investing part of his wealth in a grow strategy.

- When asset process follow GBM closed end solutions for optimal portfolio choice can be derived. From those solutions it can be deduced that when confronted with gains a loss averse investor behaves similar to a portfolio insurer aiming to retain wealth above the aspiration level. Conversely when confronted with losses the investor maximizes the probability that terminal wealth exceeds the aspiration level. This behavior is denoted as a break-even effect. It can e demonstrated that the investor becomes risk averse again for large losses as the likelihood of break-even becomes very small.
- In presence of Skewness and kurtosis (i.e not normally distributed returns) break even effects are magnified, and therefore investors would be willing to allocate more aggressively in stocks in order to reach their reference points. Highly skewed and leptokutotic return distributions are analogous then to gambles that enable the investors to either catch up with losses or reach some additional potential.

4. CHARACTERISTICS OF PORTFOLIOS CHOSEN UNDER THE BEHAVIORAL APPROACH

a. Diversification according to investment Goals

Diversification has remained as one of the main legacies from modern Portfolio Theory. The principle has proven to be not only logic and compelling but also attractive in terms of mathematical implementation. In spite of the advantages of this maxim, some investors overlook its benefits by constructing portfolios whose diversification levels are below theoretical standards. In deed, a large body of evidence suggests that investors diversify their portfolio holdings much less than is recommended by normative models of portfolio choice.

Behavioral Finance theorist have identified suboptimal diversification rules and have grouped them under the denomination of "Naïve Diversification". In parallel they have documented the existence of a "Home Bias", a tendency to bias portfolios toward securities of known companies or sometimes companies operating in an industry or geographical location familiar to the investor. Finally, studies of allocation decisions in 401(k) plans have found a strong bias towards holding own company stock: over 30% of defined contribution plan assets in large U.S. companies are invested in employer stock, much of this representing voluntary contributions by employees. Although this phenomena is related to single investors, Jorion (1994) reports that the levels of international diversification in the portfolios of big asset management companies and international financial institutions did not start to grow steadily until the end of the 80's.

In a very interesting paper Statman (2003) synthesizes the interpretation of diversification under a Behavioral Portfolio approach: "We argue that investors fail to diversify their stock portfolios because they consider individual stocks in their portfolios as the equivalent of individual lottery tickets and do not diversify among stocks for the same reason that they do not diversify among lottery tickets. A few stocks, like a few lottery tickets, provide a chance for great riches but a well-diversified portfolio of stocks, like a well-diversified portfolio of lottery tickets, guarantees mediocrity. Neither lottery buying nor undiversified portfolios are consistent with mean-variance portfolio theory but both are consistent with behavioral portfolio theory"[8]

This idea is congruent with the concepts in Prospect Theory and SP/A theory: in general investors fail to diversify not because they are not aware of the benefits of diversification (which are a widespread concept in today's markets) or because the cost and difficulty of maintaining a well diversified portfolio are out of their reach (they could easily buy shares of an Index fund that resembled the performance of a stock index of preference), they fail in the task of building diversified portfolios because they visualize their portfolios as means to achieve their aspiration levels more easily. In other words, they choose riskier(i.e undiversified) portfolios because they provide a higher probability of reaching their financial goals, not necessarily because they like risk.

The Behavioral perspective of portfolio theory suggests then that diversification is an strategy more than a rule. A well diversified portfolio would be a fair choice for a loss averse investor, however once the security goal has been attained, the same individual could invest a proportion of its wealth in alternative portfolios less diversified or highly leveraged such as those including derivatives.

b. Buy/Sell Decision and the Disposition effect

An important decision in portfolio selection is the acquisition or disposition of an asset, not only because within the context of classical portfolio theory this transaction implies a total rebalance of the portfolio but also because depending on the timing of the decision, the returns of a not well diversified portfolio might get

[8] Statman, M., "Behavioral Portfolios: Hope for Riches and Protection from Poverty", *Pension Research Council Working Paper*, 2003.

considerably affected. Several studies find that investors are reluctant to sell assets trading at a loss relative to the price at which they were purchased, at the same time individuals are more likely to sell stocks which have gone up in value relative to their purchase price, rather than stocks which have gone down. The phenomenon has become known as "disposition effect" and it was firs documented by Shefrin and Statman (1985).

What type of intuition can give an explanation to these behavior? If according to Prospect Theory people will take more risk to avoid losses than to realize gains, then it seems to be clear that the effect stems from investors with Prospect Theory-type preferences that are unwilling to realize losses by selling bad performing stocks but at the same time are willing to sell their winning stocks too fast. Thus, the investor behavior inherent in the disposition effect may be behind a puzzling feature of the cross-section of average returns, namely momentum in stock returns. In effect, the concavity of the value function in the region of gains may induce investors to sell a stock which has earned them capital gains on paper. The selling pressure that results may initially depress the stock price, generating higher returns later. On the other hand, if the holders of a stock are facing capital losses, convexity in the region of losses means that they will only sell if offered a price premium; the price is therefore initially inflated, generating lower returns later.

Behavioral literature has reported how the disposition effect does not only affect non-professional agents. The behavior of professional traders in the Treasury Bond futures pit at the CBOT reveals that this bias could be related to the mental follow up of the daily P&L. In this way, if the gains and losses of prospect theory are taken to be daily profits and losses, the curvature of the value function implies that traders with profits (losses) by the middle of the trading day will take less (more) risk in their afternoon trading.

c. Avoiding Destructive trading

The disposition effect as well as the lack of diversification can have a serious impact on the portfolio of an agent who is not aware of them. In effect if one considers the implications of unplanned buy and sell decisions and their effects on the structure of a portfolio as a whole(changes in the variance covariance relations), the benefits of an investment strategy can be reduced via increment of trading costs and loss of profit opportunities derived from early disposition.

One of the clearest predictions of rational models of investing is that there should be very little trading. In a world where rationality is common knowledge, however the volume of trading on the world's stock exchanges is very high. Furthermore, studies of individuals and institutions suggest that both groups trade more than can be justified on rational grounds. For instance, Barber and Odean (2000) examine the trading activity from 1991 to 1996 in a large sample of accounts at a national discount brokerage firm. They find that after taking trading costs into account, the average return of investors in their sample is well below the return of standard benchmarks. Put simply, these investors would do a lot better if they traded less. The underperformance in this sample is largely due to transaction costs.

In sum excessive trading derived whereas from disposition effect of or timing strategies has proven to be a sub optimal course of action that can hurt portfolio performance in the medium and long runs.

d. Skewness and Kurtosis Control

Due to the implementation of alternative models of rational choice, the Behavioral approach to portfolio theory gives more importance to the consideration of risk measures different to the standard deviation. The basic idea behind this consideration is that volatility fails as a proxy of risk because volatility per se is simply a kind statistical probability factor that does not tell the agent nothing about risk until coupled with a consequence. Te real risk of holding a portfolio is that it might not provide its owner, either during the interim or at some terminal date or both, with the cash he requires to make essential outlay. According to this point of view, the central idea is that variability should be studied in reference to some benchmark or some minimum rate of return that the investor has to exceed. Sometimes the degree to which a volatile portfolio is risky depends on what we are comparing it with. Some investors, and many portfolio managers, do not consider a volatile portfolio risky if its returns have little probability of ending up below a specifies benchmark.

In this context Skewness, Kurtosis and alternative measures of risk are used as additional sources of information and selection criteria. The attractive feature is that this type of analysis can be carried out not only to identify securities that are adequate for loss protection purposes but also for securities with high potential of superior returns.

CONCLUSIONS AND FINAL REMARKS

Just as the incorporation of expected utility as a model of decision making under uncertainty in classical portfolio theory was related to the solution the Friedman-Savage puzzle and the assumption of rational agents, the implementation of prospect theory and SP/A within the Behavioral portfolio theory models is motivated by the empirical inconsistencies of expected utility and the proven existence of limits to rationality as defined in the classical theory.

Up to date the literature seems to agree on the use of a new approach of choice under uncertainty, additionally the following characteristics can also be mentioned:

- Most of the models use one or more of the alternative theories of choice under uncertainty, plus a set of behavior biases in order to define a conceptual framework that guides the process of portfolio construction.
- Usually the risk preferences are described in terms of investor's beliefs and economic goals, this approach is congruent with a the idea of an N-fund separation theorem according to which investors allocate their wealth according to the predominance of motivations such as loss aversion and the necessity of security or the search of potential. Under this perspective the multidimensional conception of risk conduces to a different solution of the portfolio problem.
- The models assume investors that use points of reference and levels of wealth in their interpretation of the task of portfolio selection and allocation. An idea that is congruent with the concepts of mental accounting and narrow framing.

The research agenda for this field of finance is closely related to the empirical testing of the models described in this paper. A comparison of the models presented in this paper among them and against portfolios constructed under the classical approach might shed additional light on the advantages of setting up portfolios that avoid the biases discovered by Behavioral Finance theorists. It is important however to understand that an alternative approach to portfolio allocation can only be valued in terms of its contribution to investors so that they can allocate their resources in a more efficient way that actually reflects their preferences, goals, necessities.

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