Empirical Shape Function of the Limit-Order Books of the USD/COP Spot Market

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1 Introduction

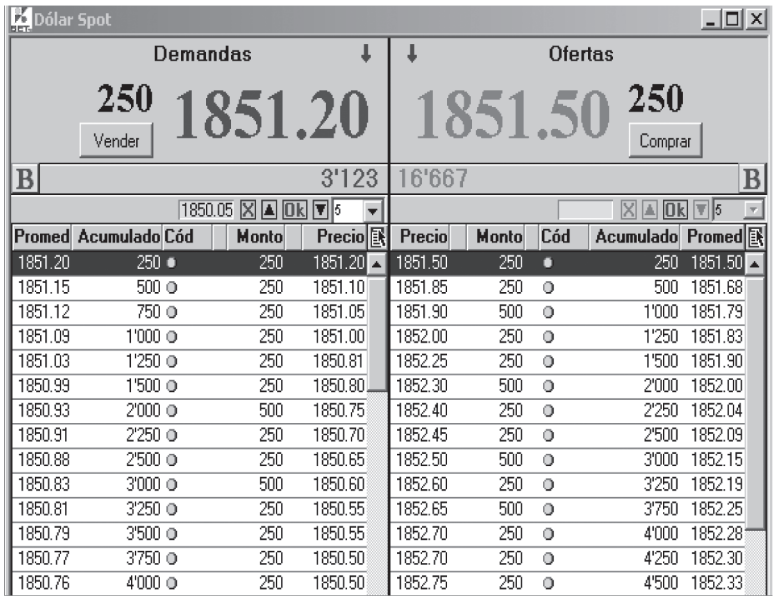
Every exchange market is organized as a dealership or a limit order market. Current markets share features from both categories. However, it is possible to identify markets with a dominant side. In a dealership market, a customer's order is filled at a single price quoted by a specialist, a institutional investorthat is inchargeofproviding liquidity. The price that the specialist provides for a certain quantity of the corresponding asset, does not affect the price that he/she would quote for different quantities. Indealership markets, dealers are expected to quote prices that clear the current buy and sell pressure. Examples of markets organized as dealership markets are Nasdaq and LSE.

In contrast, several exchange markets, including the EURONEXT and the Colombian stock and foreign-exchange markets, are organized as what is called, limit order markets, markets that do not have a specialized dealer. In limitordermarkets, participants carry on their trades submitting either limit or market orders.This differentiation isimportant because is directly related to the cost assumed by the agent when executing a buy/sell transaction.

A limit order is defined as an order that has to be fulfilled at a specific volume and price. If market conditions are not adequate for executing a limit order, the latter will be queued in what is called the limit order book until execution or cancelation, whichever occurs first. Orders in the limit order book are organized by price, with the best prices on top. It means the buy order with the highest price on the buy side of the book and the sell order with the lowest price on the sell side of the book. If multiple orders have the sameprice,they are organizedby time of arrival following a FIFO (First In First Out) mechanism.

Figure1shows a snapshot of the limit order book for the colombian exchange market on February 2nd 2011 at 10:20:09 a.m extracted from the Set-Fx, the interbank exchange platform. Suppose a market agent wants to sell 750 thousand dollars at 1852.32. She has to use a limit order. The later will be placed on the sell (right) side of the limit order book right after the order to sell 250 thousand dollars at 1852.3 and before the order to sell 250 thousand dollars at 1852.33. Before the new limit order could be fulfilled, orders in front of it have to be executed or canceled.

Figure 1: Snapshot of the limit order book on February 2nd 2011 at 10:20:09 a.m extracted from the Set-Fx, the interbank exchange platform. The best fourteen prices and volumes are presented. First column shows average prices, second column shows cumulative volume, third column shows order volume andforth



Promed	Acumulado	Cód	Monto	Precio	Precio	Monto	Cód	Acumulado	Promed
1851.20	250		250	1851.20	1851.50	250		250	1851.50
1851.15	500		250	1851.10	1851.85	250		500	1851.68
1851.12	750		250	1851.05	1851.90	500		1'000	1851.79
1851.09	1'000		250	1851.00	1852.00	250		1'250	1851.83
1851.03	1'250		250	1850.81	1852.25	250		1'500	1851.90
1850.99	1'500		250	1850.80	1852.30	500		2'000	1852.00
1850.93	2'000		500	1850.75	1852.40	250		2'250	1852.04
1850.91	2'250		250	1850.70	1852.45	250		2'500	1852.09
1850.88	2'500		250	1850.65	1852.50	500		3'000	1852.15
1850.83	3'000		500	1850.60	1852.60	250		3'250	1852.19
1850.81	3'250		250	1850.55	1852.65	500		3'750	1852.25
1850.79	3'500		250	1850.55	1852.70	250		4'000	1852.28
1850.77	3'750		250	1850.50	1852.70	250		4'250	1852.30
1850.76	4'000		250	1850.50	1852.75	250		4'500	1852.33

column shows orderprices. Demandas, ofertas, monto,precio stand for bids, offers, quantity and price respectively.

In contrast to limit orders, market orders define a specific volume and no price at which they have to be executed. Therefore, market orders take their price from the limit orders needed to be fulfilled. Now, assume that an agent delivers a market order to sell 750 thousand dollars. Because the agent's order is a market order, it will be matched with best limit orders on the corresponding limitorderbook side.If themarket or derisabuy (sell) order, it will be executed using the limit order book sell (buy) side. Therefore, the previous market order will be fulfilled using the first two limit orders of the book buy side. The selected limit and market orders leave the market as executed orders and the limit order book has changed.

Depending on which type of orders participants use, transaction costs will be different. Observing figure 1, if an agent wants to buy 250 thousand dollars, she has to choose between a market or a limit order. If she decided to use a market order, the execution price would be 1851.5. If she decided to use a limit order, the best she would have done is to place an order on the top of the buy side book, for example, selecting 1851.21 as the limit price. Assuming thatthe limitorderisexecuted (remember that the latter is not guaranteed), it will be a difference of 29 cents between both transactions. Assuming an order volume of 250 thousand dollars, the monetary difference is 72.5 thousand colombian pesos.

2 The limit order book characterization

It is possible to enumerate several features of the limit order book. ForExample, Bouchaud et al. (2008) hightlighted the fact that the limit order book is very sparse and contains few orders that are far from best quotes. It does not necessary meanthatthere are not agents willing to trade at missinggaps on the limit order book but, they may not show their intentions yet. They may see no point in revealing their intentions if prices are far from their desired value.

Another feature is the presence of iceberg limit orders, orders that are partially shown to the market in terms of its quantity size. Therefore, the limit order book shows hidden liquidity. For example, assume for illustration purposesthatthesecondquote inthebuysideof the orde rbook shown in figure 1, is an iceberg order with total size of 10 million dollars. If it came a sell market order for 4 million dollars, that market order would be cleared using just the first two orders in the book. However, when originally posted, the order was expected tobe cleared using the whole visible book.Iceberg orders avoid the agent who places it, revealing all its available information to the market.

There is also what is called a bilateral credit allocation process. itconsistson defining the maximum exposure that every market participant would like to have with every agent in the market. There could be agents that define zero credit to others. Therefore, agents have a different available limit order book based on their credit allocation. One of the main consequences of this particular feature is the occurrence of out-of-market limit orders. When agents execute market orders, their orders will match credit-approved counterpart limit orders. Thus, if executed market orders are large, there may be limit orders left that differ considerately from their neighbors in terms of offered price. In the Colombian exchange market, credit-approved limit orders are shown with a green spot. Despite their condition, non credit-approved limit orders are shown to all agents. This is a common configuration on foreign-exchange markets.

International evidence shows that the most probable final state of a limit order is cancellation. Following Gideon (2001) and Challet and Stinchcombe (2001), more than 40% of limit orders are cancelled within 10 seconds of beingplaced. More over, more than 80% of total limit orders are not fulfilled but cancel. This cancellation activity seems to decrease when the limit order's distance increases from the best quotes, Bouchaud et al. (2002).

Additionally, the limit order book dynamics have been studied thoroughly. For example, Ranaldo (2004) has found that agents' aggressiveness, agents' willing to

wait until their orders are executed, is related to the shape of the limitorderbook. Thus, limit order dynamics are directly related to timing on market participants' strategies. Large traders prefers to submit divided buy (sell) limit orders when the sell (buy) limit order book side is thin. In contrast, a thick order book motivates market orders. See for example Farmer et al. (2003),Weber and Rosenow (2004) and Weber and Rosenow (2005).

Bouchaud et al. (2002) found evidence of a power law distribution on order prices distributed around best Bid-Ask quotes. It seems that this phenomenon isrelated toagents' beliefsabouthowprobableare largejumps in current prices. The more market participants think that large fluctuations are possible, the more they will place orders far from best quotes trying to take advantage of large market orders. This behavior is normally related to how much investors value the execution speed. For example, portfolio managers, taking decisions on the long runcanbe lessconcerned about execution speed than arbitrageurs willing to tradeprice misbalances as fast as possible.

Challet and Stinchcombe (2001), Maslov and Mills (2001),Weber and Rosenow (2005), Eisleretal. (2007) and Bouchaud et al. (2002) found that the sell and buy sides of the limit order book tend to be symmetric. Also, they reported that the limit order book usually has a maximum away from the best orders (highest bid, lowest ask). The maximum away point is aprice level at which, offered volume is maximum. This results are observed on stocks intraday series of the Nasdaq Stock Market.

Biais et al. (1995) provided evidence that best quote spread and market participant actions tend to be related. When the best quote spread widens, Agentsaremore likelytoplace limitorders.The opposite also applied. Thus, small Bid-Ask spread means high liquidity and more agents placing market order instead of limit orders.

The limit order book also exhibits particular micro-patterns, which are not sufficiently explained by the current literature. Forexample, Guetal. (2008), described particular periodic peaks on the offered volume of the limit order book. According to Dorogovtsev et al. (2006), this behavior might be due to people's preferences over certain numbers like 5, 10 or their multiples. Thisfeature wasfound on stocks registered on the Shenzhen Stock Exchange (SZSE). These periodic peaks do not seem to be a local feature. Eisler et al. (2007) reported the same phenomenon on stocks traded on the London Stock Exchange.

The limit order book also shows intraday pattern. Gu et al. (2008) presented the periodogram of 1-min averaged volumes at the first three tick levels of the limit order book. It showed peaks above the noise level at $1, 2, ...day^{-1}$, which implies a period of one trading day. It is also reported an unusual increasingpatternontheav-

erage volumesof the first tickswhich contrast with the regular U or L shape found in returns, return volatility, trading volume orbid-ask spreads. For instance,see Brooks et al. (2003) and Tian and Guo (2007).

Next sections studies the USD/COP limit order book and its empirical and statistical properties.

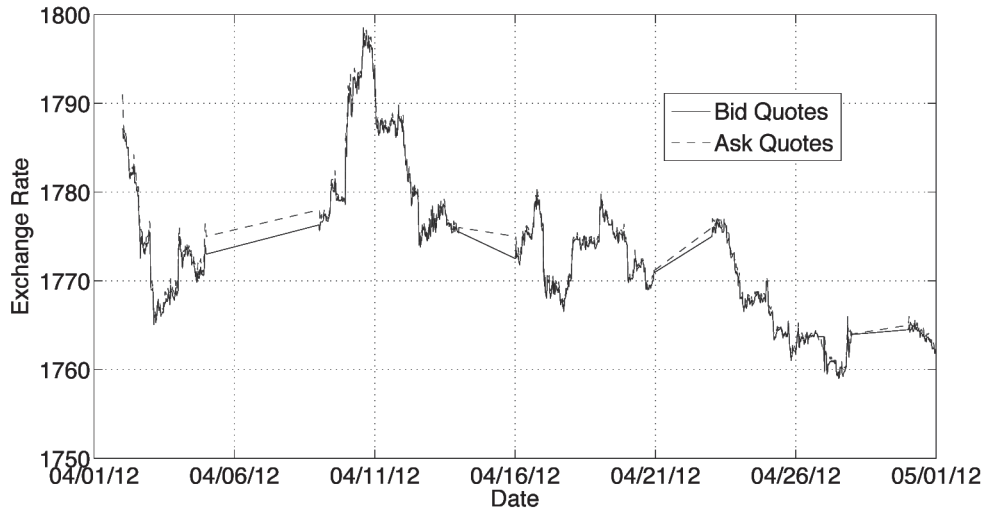
3. DataSet

In order to extract empiral features from the USD/COP limit order book, this paper will use a dataset consisting of one month of tick-by-tick information from the Set-Fx platform, the OTC market where USD/COP spot is negotiated.Fromtick-by-tickdata, limit or derbook (LOB) hasbeenreconstructed. Book insertions refer to order creations. On the other hand, order book extractions consist of cancelled or matched orders.

The dataset as a whole, consist of one hundred thousand market messages that have been organized to obtain corresponding USD/COP limit-order book. Dataset corresponds to April 2012 market data.

Except when stated, this paper uses event time, instead of clock time. Thus, all time series will be constructed indexed to orderevent arrival. Mainevents are insertion, cancellation, modification and matching. Figure 2 shows Bid-Ask quote prices during dataset period. USD/COP exchange rate located between 1.760 and 1.800 Colombian pesos.

Figure 2: Bid-Ask quote prices for April 2012.



4 Limit-Order Book Averaged Shape

Due to the large amount of information, the first efforts have been concentrated on averaged shape LOBS. The averaged shape of the Bid or Ask limit-order book can be calculated as follows:

$$V_{b,s}(\Delta) = \frac{1}{M}\sum_{t=1}^{M} V_{b,s}(\Delta, t), \tag{1}$$

$$\Delta = \begin{cases} \frac{(p_b - p)}{u} + 1 & \text{for bid orders.} \\ \frac{(p_b - p)}{u} + 1 & \text{for ask orders.} \end{cases} \tag{2}$$

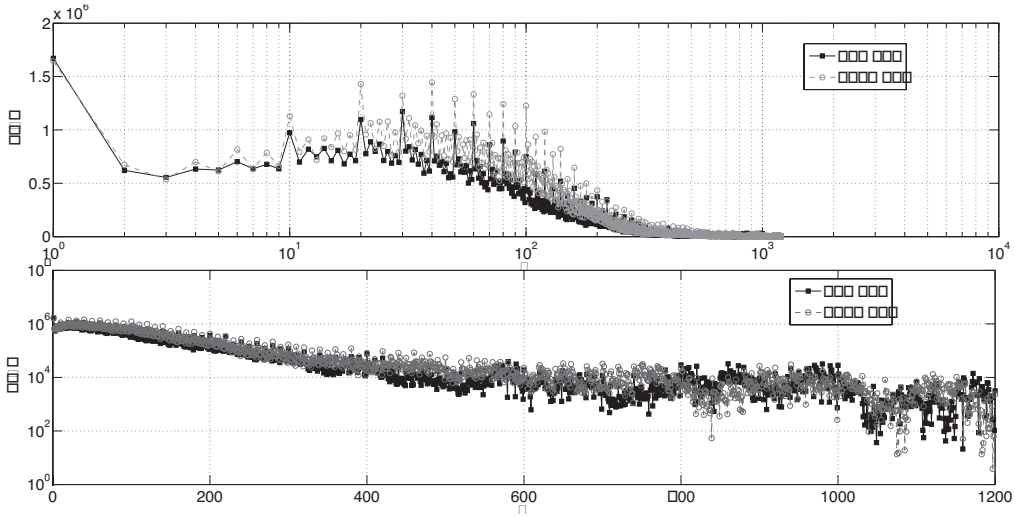
M is the number of total events in the dataset and $V_{b,s}(\Delta, t)$ is the Buy(b)Sell(s) volume at certain instant t and Δ. Δ is the distance from best Bid-Ask prices. The Δ of best Bid-Ask prices is 1. u is the minimum price variation set equal to 0.05 Colombian pesos.

As observed on upper figure 3, Ignoring best quotes, the averaged shape LOBS show a maximum away at $\Delta_{max} = 20$. This maximum away has been also observed in other markets as in Bouchaud et al. (2002) and Gu et al. (2008). LOBS increase when $\Delta < \Delta_{max}$ and decrease afterwards. However, the averaged shape LOBS show high volumes at best quotes. This finding is unique for the USD/COP spot market and could be interpreted as evidence of participant's aggressiveness. It may be that execution probability is more relevant than timing.

Another interesting feature, is the presence of periodic peaks at $\Delta = 2n$ for $n = 1, 2, ..., 10$ and $\Delta = 10n$ for $n = 1, 2, ...$ In terms of Colombian pesos, these periodic peaks correspond to10 and 50-cent intervals respectively. Market participants tend to prefer some price levels to others when inserting limit orders. Specially, participants prefer round prices to 10 cents when $\Delta \leq 10$, and round prices to 50 cents otherwise. This preference phenomenon is also observed on stocks from the London Stock Exchange, Eisler et al. (2007).

Different from Gu et al. (2008), the USD/COP LOBS do not abruptly decrease at some Δ. This phenomenon can be explained by the absence of circuit breakers on the USD/COP spot market.

Figure 3: Upper: Averaged shape Bid-Ask LOB in log-linear scale.
Lower: Averaged shape Bid-Ask LOB in linear-log scale.



Lower figure 3 shows that average volume seems to decrease exponentially. Therefore, assuming that

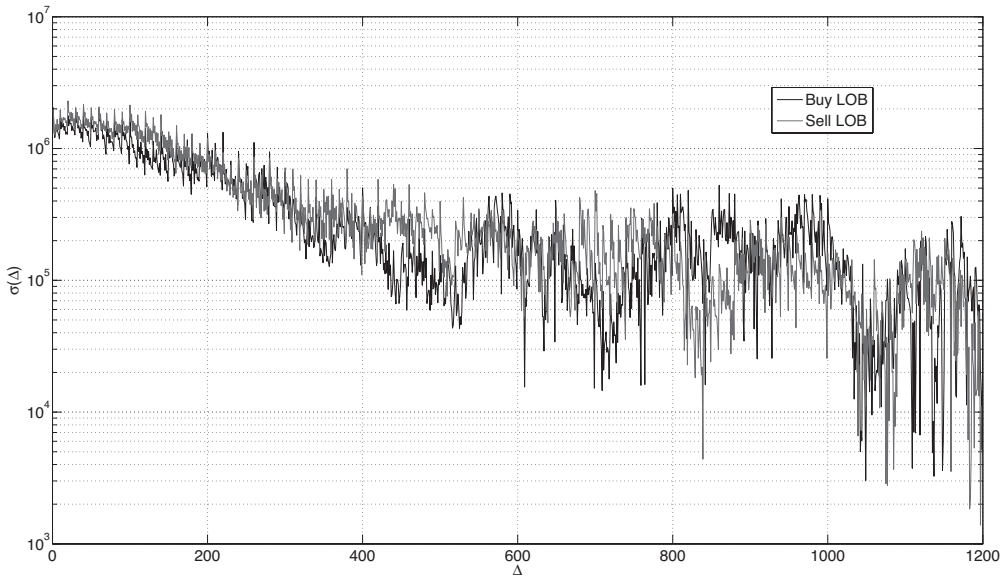
$$V_{b,s}(\Delta) \sim \exp^{-\beta_{b,s}\Delta} \tag{3}$$

and using a least-square fitting method, it is obtained βb =0.0051 and βs = 0.0055. There is no significant difference between both β parameters. Therefore, comparing the calculated βs to the ones obtained in Gu et al. (2008) for the Chinese Stock Market, there is no evidence that USD/COP participants use differentiated strategies for long and short positions.

Though the averaged shape LOB is a good starting point, limit order volumes can largely deviate from their means. Therefore, it is necessary to analyze fluctuations of volumes using the standard deviation, σ, as a function of different values of Δ. That is,

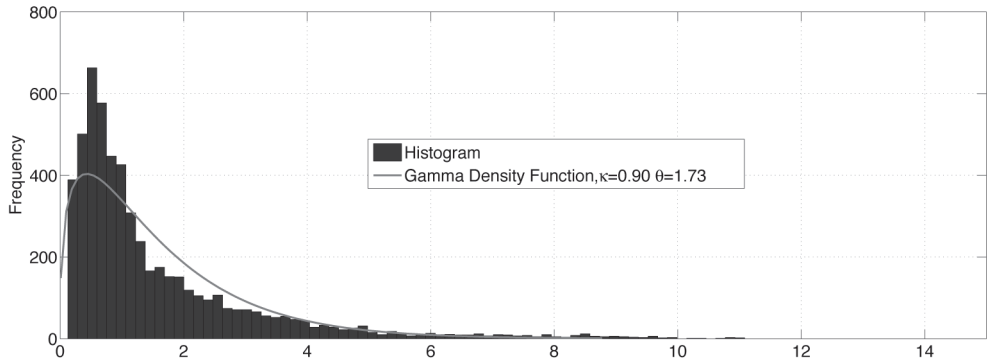
$$\sigma_{b,s}(\Delta) = \sqrt{\langle V_{b,s}(\Delta)^2 \rangle - \langle V_{b,s}(\Delta) \rangle^2}. \tag{4}$$

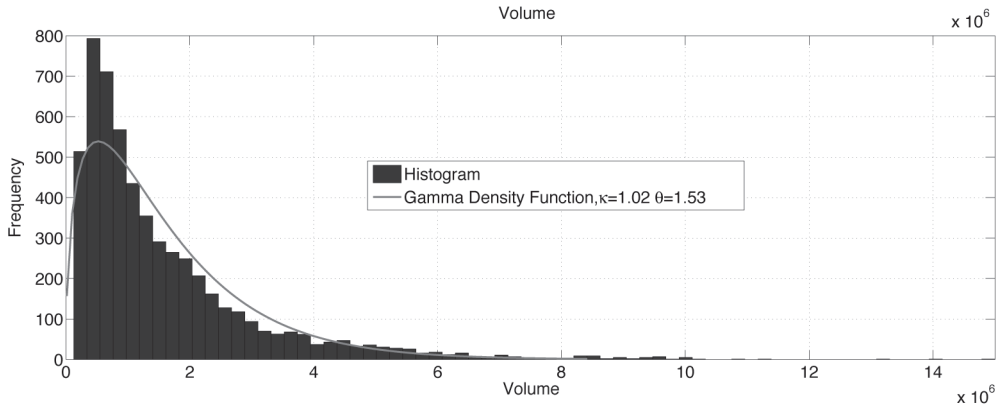
Figure 4: Averaged Shape Bid-Ask LOB standard deviation in linear-log scale.



The standard deviation for Bid-Ask LOBS are shown in figure 4, volume standard deviation are inversely related to Δ. However, standard deviation become more volatile when $\Delta > 500,25$ colombian pesos. Limit order volume is less disperse up to some Δ. Deep orders are too unstable. This behavior is observed in both LOBS.

Figure 5:Upper:1-min averaged volume histogram at $\Delta = 1$ for the Bid LOB. Solid line corresponds to the best-fitted gamma density function. Lower: 1-min averaged volume histogram at $\Delta = 1$ for the Ask LOB. Solid line corresponds to the best fitted gamma density function.



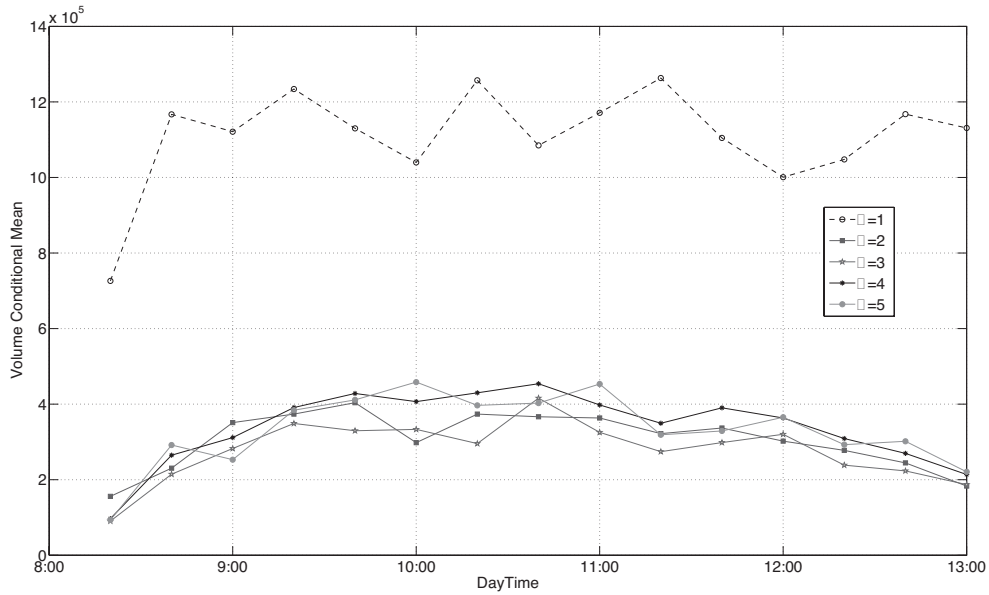


5. Statistical Properties of Volumes at vidual Tick Levels

5.1 Probability Distribution

Previous section concentrates on averaged shape LOBS and their standard deviations. However, in order to study statistical properties of limit order volume, it is necessary to calculate time averaged volume over fixed time intervals, δt, at individual price levels.

Figure 6: Intraday volume conditional mean at Δ values ranging from 1 to 5.

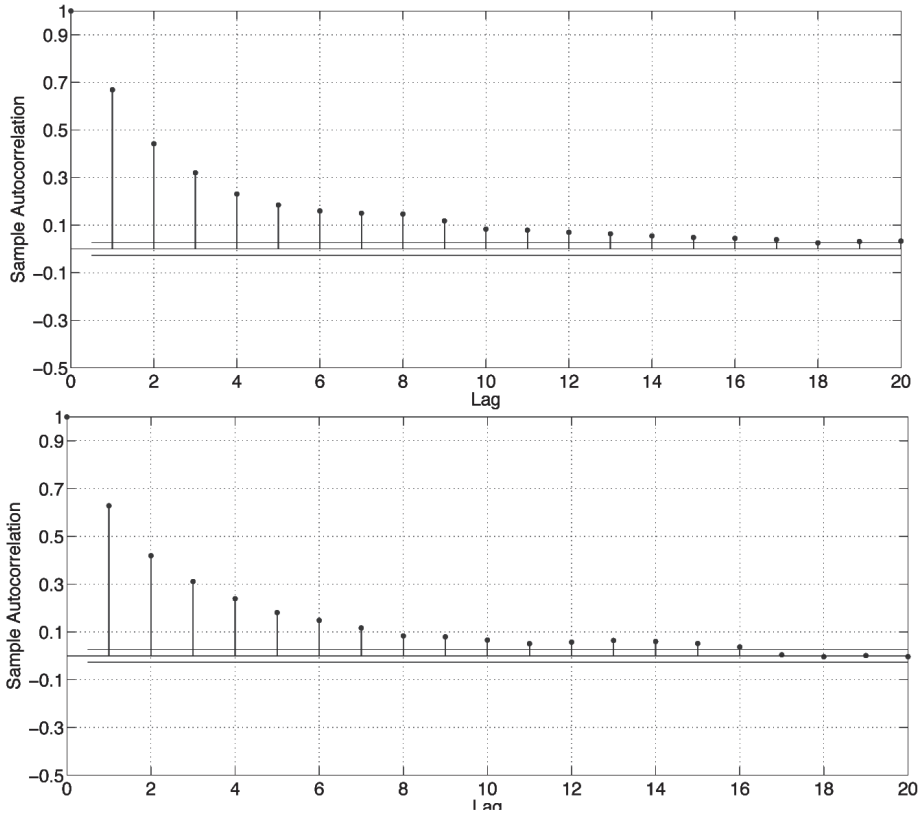


$$\upsilon_{b,s}(\Delta, t) = \frac{1}{N} \sum_{i=1}^{N} V_{b,s}(\Delta, t_i), \tag{5}$$

where t_i is the time moments of the N events that belong to the interval $(t - \delta t, t]$. N is a function of t and δt. δt has been defined as 1 minute.

Differentfrom Gu et al. (2008) and in line with Bouchaud et al. (2002), 1 min averaged volume series are best represented by a Gamma distribution. Figure 5 depicts the probability distribution parameters for Bid and Ask LOBS at $\Delta = 1.1$-min averaged volume series at higher values of Δ are also well represented by Gamma distributions. As figure 3 will predict, distribution mean decreases as Δ increases.

Figure 7: Upper: 1-min averaged volume autocorrelation function calculated over Bid LOB. Lower: 1-min averaged volume autocorrelation function calculated over Ask LOB.



5.2 Intraday Pattern

Intraday patterns are well documented on financial time series including return, volatility, volume and duration series. See for example, Andersen and Bollerslev (1997), Andersen and Bollerslev (1998), Kiymaz and Berument(2003), Brooks et al. (2003) and Tian and Guo (2007). Though every study uses different market data and time frames, almost all papers come to the same conclusion. There exist inverted U or L shape patterns on financial intraday series.

Figure 8: Upper: Hurst Coefficient calculated over the 1-min averaged Bid-Ask LOBs. Lower-Left: Detrended fluctuation function calculated over the Bid LOB for Δ = 1. Lower-Right: Detrended fluctuation function calculated over the Ask LOB for Δ = 1.

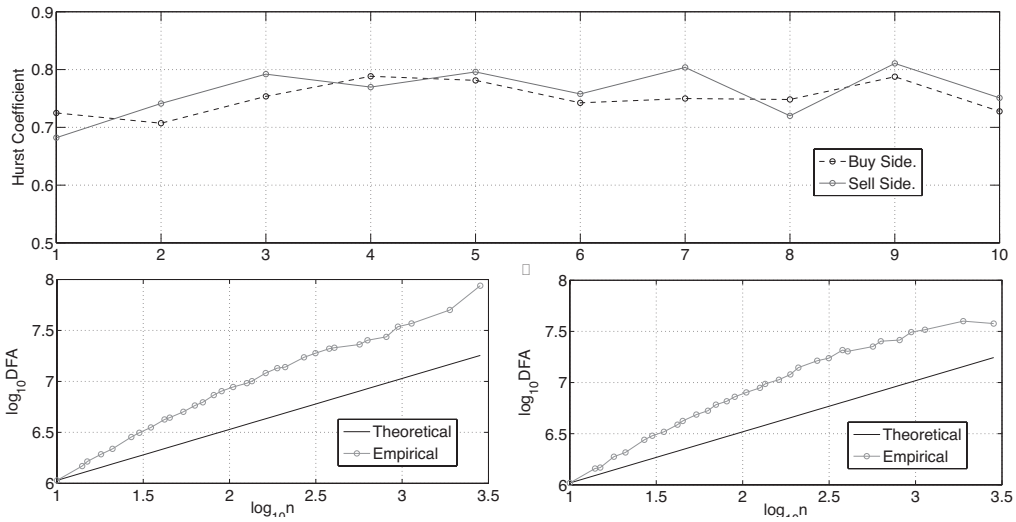


Figure 6 shows the volume conditional mean at Δ values which range from 1 to 5. At Δ = 1, there is a significant difference between volume before and after 8:30 am. For Δ > 1 volume intraday conditional means resemble an inverted U with maximum value around 10 –11 am.

5.3 Dependency and Long Memory

Temporal correlation on time series can be assessed through the autocorrelation function. Using the autocorrelation function shape, it is possible to determine what kind of dependency is observed on data. For example, it is interesting to

observe function's decaying speed if any autocorrelation coefficient locates out of the regular 95% confident level interval. As shown in figure 7, autocorrelation coefficients are significant up to Lag 16. Therefore, it may be evidence of long memory in the series.

In order to confirm the long memory feature, the Hurst Coefficient has been calculated for the 1-min averaged volume series using DFA, a popular method. DFA analysis has been performed on both, Bid LOB and Ask LOB for Δ ranging from 1 to 10. Figure 8 shows results. In line with the low decaying speed of the autocorrelation function, all Hurst Indices, which values locate between 0.7 and 0.8, support high persistence levels. Hurst Coefficients were also calculated for $\Delta > 10$. However, the increasing presence of 0 values drives indices to 1.

6. Conclusion

The empirical shape of the Bid-Ask LOBS has been investigated. Some findings confirm the international evidence others are unique for the USD/COP spot market.

As other research papers highlighted, the USD/COP averaged volume LOBS also show maximum away values and periodic peaks. However, due to the absence of circuit-breaker prices, volume do not abruptly decay at some value. In contrast, volume does not decay but become volatile at high Δ values. There was not significant difference between Bidand Ask LOBS which weakens the differentiated Bid-Ask strategy hypothesis.

USD/COP1-min averaged volume probability distribution was greatly represented by a gamma distribution. Distribution mean decreases as Δ increases.

1-min averaged volume series exhibited in traday patterns.When $\Delta = 1$volume were significantly different before and after 8:30 am. For other Δ values, an Inverted U was regularly observed. The volume series also showed a high persistence level confirmed by the series' autocorrelation function and the Hurst Coefficient.

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