A CVA approach for financial institutions in Colombia

Una aproximación al CVA para instituciones financieras en Colombia

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Abstract

This document explains the components of Credit Value Adjustment (CVA), based on theoretical models and presents ideas for its implementation by financial institutions in Colombia using common computer tools.

Key words: Credit value adjustment; valuation, derivatives.

JEL classification: G13, G20, C60.

Resumen

En este documento se explican los componentes del ajuste a la valoración por riesgo de crédito (CVA, por sus iniciales en inglés) a partir de modelos teóricos, y se presentan ideas para su implementación por parte de instituciones financieras en Colombia utilizando herramientas informáticas comunes.

Palabras clave: ajuste de valor crediticio; valoración; derivados.

Clasificación JEL: G13, G20, C60.

Introduction

Between 2007 and 2008, the world witnessed how trillions of dollars vanished from the financial market. The well-known "global financial crisis" hurt the world economy in ways that are still the subject of study. However, something is now clearer than ever: any financial institution can represent a threat to financial stability.

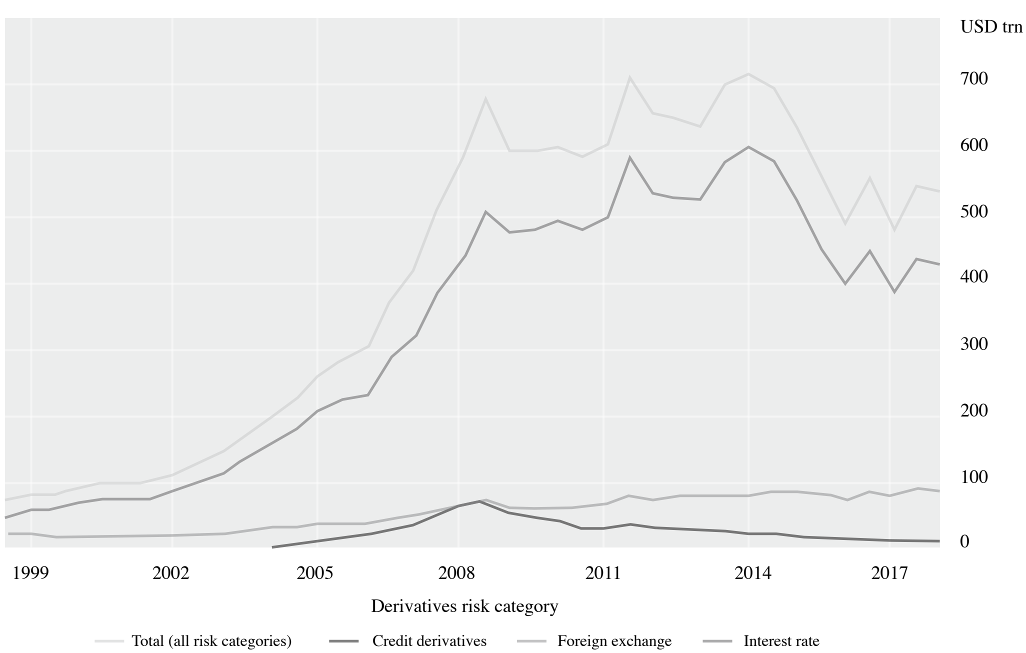
Nobody expected the bankruptcy of gigantic financial institutions, also known as the "too big to fail" companies, such as Lehman Brothers and Merrill Lynch. These Wall Street giants went bankrupt in a matter of days as Lehman filed for Chapter 11 bankruptcy[1] and Merrill sold for a symbolic share price. The failure of these institutions was a catalyst of financial destruction and the first to suffer the impact were the counterparties they traded with on a daily basis and with whom they held huge open amount positions in their portfolios. Counterparty credit risk was the main player during and after the crisis spread, and the need to quantify financial exposure was never more imperative.

1 Chapter 11 is a form of bankruptcy that involves a reorganization of a debtor's business affairs, debts, and assets. Named after the U.S. bankruptcy code 11, corporations generally file Chapter 11 if they require time to restructure their debts.

The depth of the financial crisis is still being measured, and it is a matter of global concern for the interconnected financial market.

Many studies suggest that the lack of a strong regulatory framework for OTC derivative markets was one of the main causes of the crisis development. Some of this deregulation process that took place under the Greenspan administration is detailed in James Crotty's paper "*Structural Causes of the Global Financial Crisis: A Critical Assessment of the 'New Financial Architecture'*" (Crotty, 2008). Not a single market participant was paying attention to the counterparty risk involved in OTC trading. Derivatives were freely traded, and risks were shared among all participants, including foreign investors. Because of the highly customizable features of these products, the OTC market growth had no precedence in the years before the crisis. Figure 1 shows the growth of OTC derivatives by underlying risk from 1999 to 2017 (BIS, 2018).

Figure 1: OTC derivatives notional amount outstanding by risk category



Source: BIS (2018).

As evidenced in the illustration above, under Bernanke´s administration, as Chairman of the Federal Reserve Bank, OTC derivatives grew at an exponential rate from 2005 until 2008. Bernanke, an advocate of the free-market, kept interest rates low and promoted free-market practices as well as the deregulation of derivatives. The market, as is true in nature, will find its way. As a result, the perfect environment where OTC derivatives could grow incredibly fast was set.

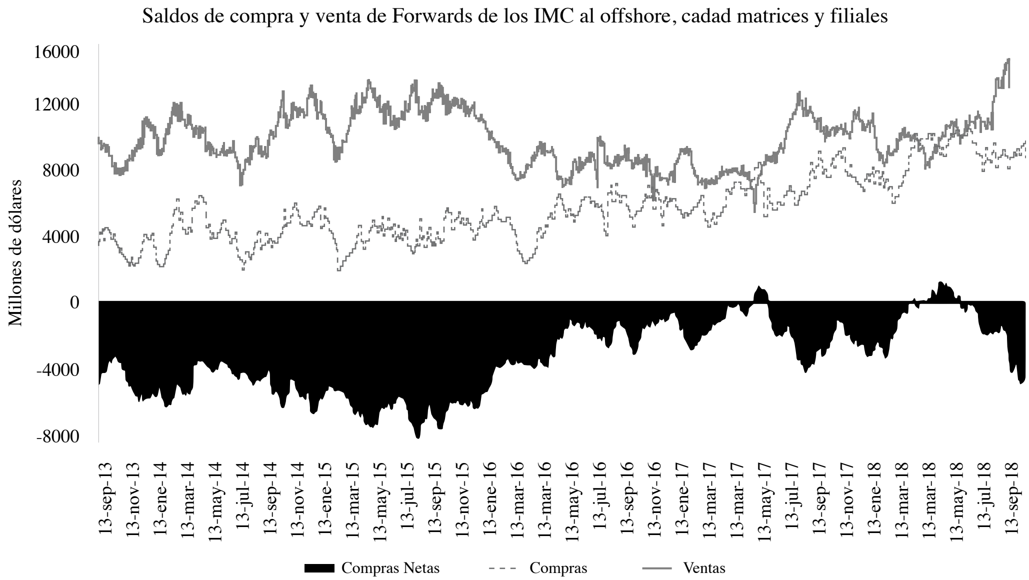
Figure 1 also shows that OTC Credit derivatives trading has decreased since 2008, this due to trade compression aimed at eliminating redundant contracts aided by a shift towards CCP trading, as mentioned in the BIS OTC derivatives statistics at the end-June 2016 report (Bank for International Settlements, 2016). However, reducing the outstanding notional amounts traded in OTC derivatives markets is not an easy task, and in the meantime, international regulatory standards will have to fill the gaps between standardized markets and the huge highly customizable bilateral market.

While efforts in shifting towards central clearings are being made, a whole new set of rules and best practices were introduced to prevent financial institutions from suffering liquidity and leverage problems. Basel III, IFRS and the US Dodd-Frank Wall Street reform embedded regulatory standards that aimed at increasing the stability of the OTC derivatives by forcing institutions to constantly measure counterparty credit risk, collateral management and CVA (Credit Value Adjustment). As long as OTC derivatives exist, there must be tools to measure counterparty credit risk among all participants.

Regarding that matter, all countries needed to move forward looking for strategies to protect their already wounded financial markets. The most developed countries were first in adopting Basel regulatory standards as the rest of the world followed. This meant of course that any nation whose financial institutions are interested in trading with these, will be asked to rigorously adopt and supervise the same set of rules. Colombia is not the exception. In fact, by 2018 the weekly report on Financial Markets released by the Colombian Central Bank shows that Colombian Financial institutions trade on average 16 billion US dollars per week just in Fx Forward contracts with offshore counterparties, Figure 2 (Banco de la República, 2018).

The impact of the crisis on Colombia should not be underestimated. The national economy experienced an important deceleration, unemployment rates increased, as exports and GDP decreased. In order to maintain the already important offshore market and aiming to strengthen the local financial institutions against future crises, the Colombian financial supervisor decided to adopt the standards of Basel III.

Figure 2: Notional amount in FX Forward contracts with offshore counterparties



In this context, understanding counterparty credit risk, and in particular CVA, has become more imperative than ever in today's Colombian financial market. During the last six years, the financial supervisor has been introducing a handful of regulatory requirements to make sure all financial institutions are judiciously monitoring their financial exposure to all their derivative counterparties, including of course those that belong to the real sector of the economy.

Few of the biggest financial institutions in Colombia were familiar with the acronym of CVA, and probably none of them have the technological tools required to price such large portfolios. In fact, most of them do not even possess the technological software platforms used to run front-to-back treasury operations. Others are just starting the long and exhausting task of implementing a treasury software capable of performing portfolio CVA calculation in real time or at least in the end-of-day process along with accounting.

Nevertheless, technological infrastructure and software implementation are not the only barriers to overcome. Many of these financial institutions do not have the correct models to run counterparty risk valuation over their portfolios, and they may be resorting to the game of implementing random models to meet the financial supervisory regulatory standards on time. It is also a regular market behaviour that when under pressure some financial institutions choose to adopt weak and less accurate models in order to meet regulator deadlines. In fact, many Colombian financial institutions have implemented CVA calculations on Excel files with little computational power for the portfolios they hold in addition to not having embedded these modules into their core accounting systems. This of course creates high operational risks and will also lead to incorrect portfolio decision making. Please note that the financial supervisor may also lack the sufficient knowledge of counterparty credit risk, and therefore may not have the tools to oversee market compliance.

Prior to 2012, all financial institutions in Colombia used internal models to price their portfolios. This, of course meant that two financial institutions that traded with each other and, therefore, were reporting and pricing the same derivatives to the local supervisor, may have had different values on their balance sheets. This situation created price distortion and probably arbitrage opportunities, besides going against market efficiency theory.

By November 2012, the Colombian financial supervisory board, the Financial Superintendence of Colombia issued the ce050 act, in which financial institutions are forced to choose a "price vendor", which is an entity in charge of providing direct prices and/or pricing supplies such as FX rates and IR curves to value their portfolios. A price vendor is not only a market data or price supplier, but also determines the models used for pricing each instrument, including OTC derivatives (Superintendencia Financiera de Colombia, 2012).

By December 2015, the Colombian financial supervisor issued the ce041 act, in which chapter 18 of the main regulatory financial standards book called "Circular Básica Contable y Financiera" had been modified including for the first time CVA in the OTC derivatives calculation (Superintendencia Financiera de Colombia, 2015). In it, the Financial Superintendence of Colombia set the rules by which financial institutions should account for counterparty credit risk in the pricing and reporting of OTC derivatives (It is assumed that standardized derivatives or central cleared instruments do not incorporate counterparty risk). The dilemma of whether or not central clearing is counterparty risk free is also discussed by Gregory, who draws attention to the dangers that arise on the use of central clearing. According to Gregory, CCPs (Central Counterparty Institutions) distribute counterparty risk instead of eliminating it, since the failure of one of the members will result in the distribution of the losses among all surviving members. In addition, this risk-sharing results in moral hazard where no institutions have any incentive to monitor any of the other member's credit quality since there is a third party taking the risk. As a result, central clearing creates other types of risk which are shared by all members and therefore add up to systemic risk (Gregory, 2012).

Finally, financial institutions in Colombia face the challenge of meeting regulatory standards of CVA and DVA (Own risk) calculation in an efficient but accurate manner, assuring that counterparty risk is controlled and properly hedged.

The aim of this research is to explain each and every component of Credit Value Adjustment (CVA) in detail based on theoretical models and show how financial institutions in Colombia would be able to implement such models in an efficient manner using common computational tools without investing in pricey treasury management systems or incurring in expensive solutions provided by price vendors. In this paper, an efficient model describes a situation in which financial institutions are able to run CVA calculations with low time consumption, high accuracy and low implementation costs. In order to show that the model executed in this document is accurate and efficient, we will run our fully theoretical CVA model using computational tools for a sample Fx Options portfolio and then compare the outcome with a simpler model used by some financial institutions. We will outline the differences and display the results

1. Theoretical framework

There are some concepts that need to be addressed first for a reader to fully understand this paper. Some are related to the mathematical concepts that underlie the model that will be discussed and some are financial concepts related to counterparty credit risk and market risk.

1.1. Random Walk

The Random Walk is a stochastic process used in many disciplines to simulate or to approximate the path of an object or variable. In finances, the concept is used to simulate the fluctuation of stock and commodity prices. It is also considered today's asset price model cornerstone. The model was introduced by Louis Bachelier in 1900 as an attempt to approximate the prices on the Paris stock exchange.

Mathematically, a random walk is a process where the current value of a variable is composed by its past value plus an error defined as white noise. In this sense, the random walk is presented as follows:

$$y_t = y_{t-1} + \varepsilon_t$$

Where y_t is the current value of the variable, y_{t-1} is the past value of the variable and ε_t is an error that follows a normal distribution with zero mean and variance one. As described above, the process implies that the best prediction of y_t is its current value. Which is to say that the process is also Martingale.

1.2. Brownian motion process

Brownian motion refers to the random movement of a particle observed at a macroscopic level in *d*-dimensional space. On the microscopic level, a particle random movement is caused by other particles hitting it or by external forces. It was named after Robert Brown who, in 1827, while doing his research on the zig-zagging motion of the particles discovered the physical phenomenon. As the Brownian motion is used to explain the physical phenomenon, the mathematical aspect of it is named Wiener process.

Figure 3: Planar Brownian motion

Source: Peres (2008).

Now, if the particle is evaluated at time zero S_0, its position at any time n in the future is given as $S_n = S_0 + \sum_{i=1}^{n} X_i$ where $X_1, X_2, X_3, \ldots$ are assumed to be independent and identically distributed variables. As a result, the process $\{S_n : n \geq 0\}$ is a random walk that follows the characteristics mentioned earlier. The Brownian motion process is described as $\{W(t): t \geq 0\}$. If the process starts in zero $W(t)=0$ then the process is known as *standard Brownian motion*.

The Brownian motion or Wiener process follows four principles:

1. Every increment $W(t)-W(s)$ over an interval $t-s$ is normally distributed with mean 0 and variance $t-s$, that is $W(t)-W(s) \sim N(0,\ t-s)$.
2. For all time intervals $[t_1,\ t_2]$ and $[t_3,\ t_4]$ with $t_1 < t_2 \leq t_3 < t_4$, the increments $W(t_4)-W(t_3)$ and $W(t_2)-W(t_1)$ are independent random variables.
3. $W(0)=0$.
4. $W(t)$ is continuous for all t.

1.2.1. Geometric Brownian motion process

Since Brownian motion (BM) as presented earlier can result in negative values, its application on stock prices not appropriate as prices can only be positive or zero, $S_t \geq 0$. As a result, a variation of the BM is presented as *Geometric Brownian motion* (GBM). Any stochastic process S_t is said to follow a GBM if it satisfies the following stochastic differential equation:

$$dS_t = \quad S_t dt + \sigma S_t dW_t$$

Where W_t is a BM process and μ is drift and σ is the percentage volatility and both are constants.

Consequently, $\mu S_t\, dt$ determines the "trend" and $\sigma S_t\, dW_t$ determines the "random noise" effect on the trajectory.

Separating and integrating both sides we have:

$$\int \frac{dS_t}{S_t} = \int \left(\quad dt + \sigma dW_t \right) dt$$

Since $\frac{dS_t}{S_t}$ implies to derivative of $\mathrm{In}(S_t)$ and applying *Itō* then we get to:

$$\ln\left(\frac{dS_t}{S_t}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t$$

Now, if applied exponentially on both sides we obtain the analytical solution of a GBM expressed by:

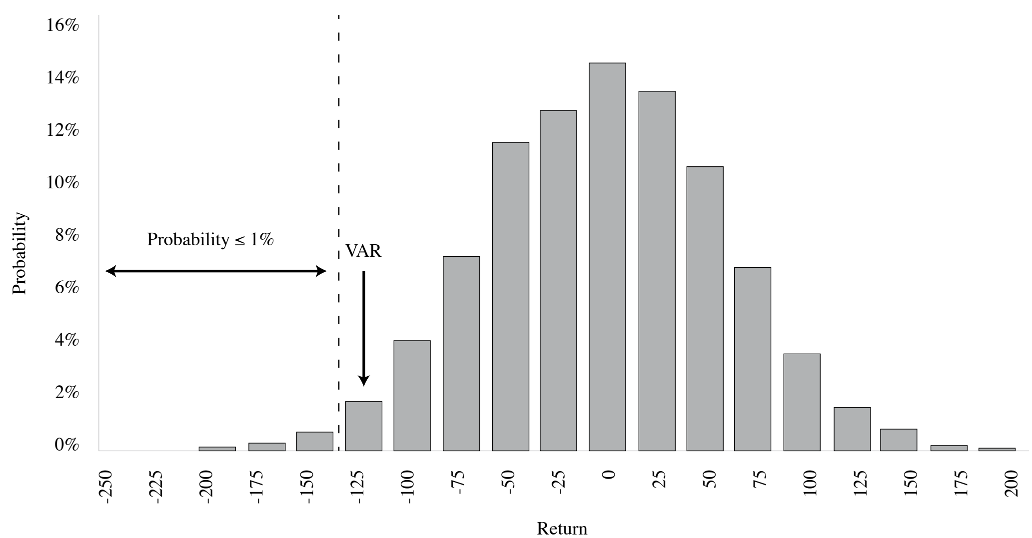
$$S_t = S_0 e^{\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)}$$

The resulting equation as GBM is used in the approximation of stock prices as it will only take positive values and has lognormal distribution $X \sim \log\text{normal}\left(\mu,\ \sigma^2\right)$.

1.3. Value at Risk (VAR)

Value at risk or most often known as VAR is a popular market risk model that aims to determine the worst loss over a target horizon at a certain confidence level $\alpha\%$. This means that the resulting value will be exceeded with no more than a $(1-\alpha)\%$ probability.

Figure 4: VAR



Source: Gregory (2010).

Figure 4 illustrates that the VAR is 125, which is the worst loss scenario at 99% confidence level.

VAR can be calculated using three basic models:

1. **Historical VAR**: The model is based on the historical data re-organized under the assumption that history will repeat itself. Knowing that returns follow a normal distribution, once the historical data is rearranged and presented in a histogram, it is possible to evidence a perfectly shaped Gauss curve. Now, under a certain confidence level of $\alpha\%$ one can find the worst loss scenario.
2. **Monte Carlo simulation**: It is said that unlike historical VAR, the Monte Carlo method for VAR does not rely in historical data to forecast future prices. As Monte Carlo is used for stochastic processes, the forecast of future prices must be expressed in terms of probability distribution. However, Monte Carlo simulations require a setting of parameters such as volatility and correlations that rely on past experience. As a result, the method generates a number of random scenarios based on the nature of the distribution, in this case Normal distribution, and its variance or volatility. Again, the resulting data is a Gaussian curve that displays the worst loss scenario at a certain confidence level.
3. **The variance-covariance method**: This method is widely used among financial institutions since it takes into account more than one product and currency. The model makes the assumption that there is correlation between the analyzed assets. Therefore, under the variance-covariance method, it is needed not only to collect historical data but the correlation between each pair of assets. In the end, the VAR is the resulting value of using the variance-covariance matrix.

1.4. Credit risk

Credit risk is the risk that a debtor of a credit contract may be unable to fulfill a contractual obligation or make a payment on the due date. From an institution's point of view, the credit risk would be the probable risk of loss resulting from a borrower´s failure to make a payment. Traditionally, this is often known as default. In terms of capital markets, a default probability will be calculated throughout the lifetime of the instrument.

1.5. Wrong way risk

The concept of wrong way risk is widely used in counterparty credit risk. It represents the correlation between the exposure at default and the counterparty default time. In other words, the Wrong Ways risk concept represents an unfavorable dependence between exposure and counterparty credit quality.

There are two ways of wrong way risk, the exposure is high when the counterparty is more likely to default, or the exposure is low when the counterparty is financially very healthy. Off course, when studying counterparty credit risk, we focus on the latter since the former does not pose a risk to the institution.

1.6. Counterparty risk

Counterparty risk is the risk that a business with whom one has entered into a financial contract will fail to make the payment or fulfill their side of the contract. According to Gregory, Counterparty risk represents a combination of market risk and credit risk. In this context, the level of exposure and the counterparty's default probability are taken into account when pricing a portfolio. It is said that a counterparty with a large default probability and small exposure may be considered preferable to a counterparty with a small default probability and large exposure (Gregory, 2012).

As opposed to lending risk, where the notional amount is known with a degree of certainty throughout the lending period and only one party takes the risk, in counterparty risk the value of the contract in the future is uncertain and, since the value can be either positive or negative the risk is typically bilateral.

1.7. Credit exposure

Credit exposure is a concept that refers to the loss in the event of a counterparty defaulting. Since the counterparty can default at any time, the exposure is definitely a time-sensitive measure. Now, exposure is conditional on counterparty default, it means that it is relevant only if the counterparty is unable to fulfill its contractual obligation (defaults).

The value of credit exposure can be *positive* or *negative*. A negative exposure will reflect that an institution is in debt to its counterparty. From counterparty risk perspective, a negative exposure is not relevant since the institution is still obliged to settle this amount; this is the case of DVA. On the other hand, a positive exposure will represent a loss in the event of counterparty default. It is assumed that the institution will still recover a fraction of the amount due (R).

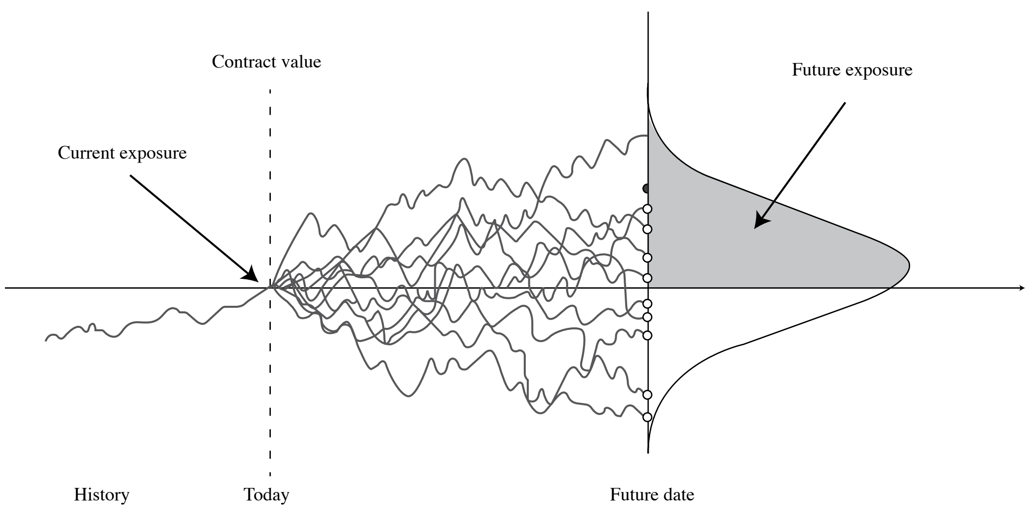
Exposure is now defined as $Exposure = \max(value, 0)$ where *value* is assumed risk-free.

Which is equal to $(\mu + \sigma Z, 0)$, where Z is a normal standard variable.

1.7.1. Future Exposure

The concept of future exposure is related to the uncertainty of the value. Whilst past and current exposures are known, future exposure is determined probabilistically by future market movements and contractual features of transactions which are uncertain.

Figure 5: Future exposure



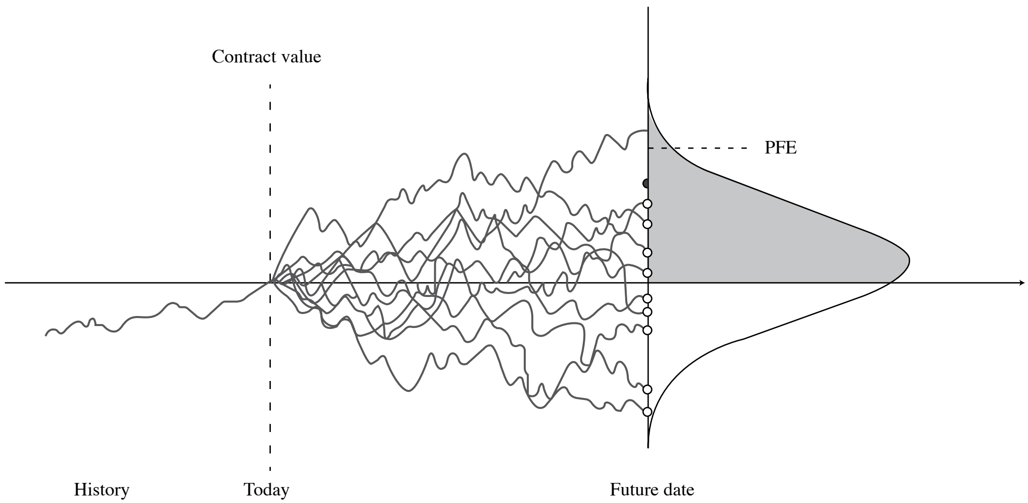
Source: Gregory (2012).

As Figure 5 shows, future exposure accounts only for positive values.

1.7.2. Potential Future Exposure

Whilst future exposure represents the positive side of all future values, *potential future exposure* (PFE) answers to the question of *what the worse exposure is that an institution could have at a certain time in the future*. Figure 6 illustrates PFE.

Figure 6: Potential Future Exposure



Source: Gregory (2010).

As it is possible to evidence, PFE calculation is exactly the same as that used for VAR. Thus, PFE is given by:

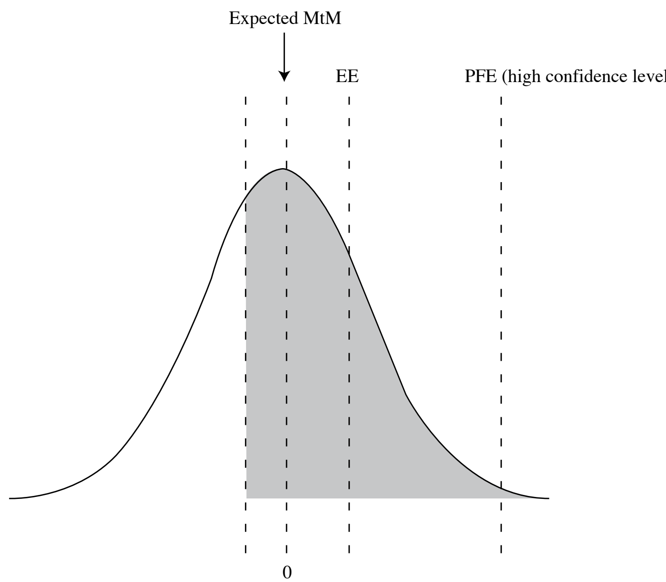
$$PFE_{\alpha} = \mu + \sigma \Phi^{-1}(\alpha)$$

Where Φ^{-1} is the inverse of a cumulative normal distribution function.

1.7.3. Expected Exposure (EE)

In addition to FE and PFE, the pricing of counterparty risk involves the calculation of Expected exposure (EE), which is the average of all positive exposures at a given time t.

Figure 7: Expected Exposure



Source: Gregory (2010).

Given that Exposure is defined as $\max(\mu + \sigma Z, 0)$, and expected exposure is the average of its positive future values, therefore EE is defined as:

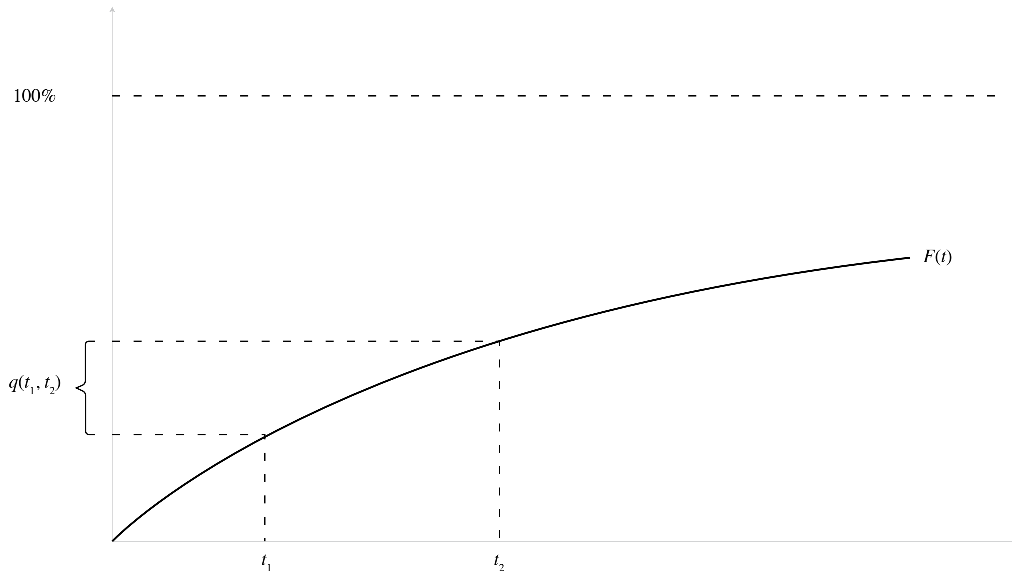
$$EE = \int_{\frac{\mu}{\sigma}}^{\infty} (\mu + \sigma x)\varphi(x)dx = \mu\Phi(\mu/\sigma) + \sigma\varphi(\mu/\sigma)$$

Where $\varphi(\bullet)$ is a normal distribution function, and $\Phi(\bullet)$ is a cumulative normal distribution function.

1.8. Default probability

Counterparty's default probability is a cumulative function $F(t)$ conditional upon no default at the current time. The function gives us the default probability at any point priori t.

Figure 8: Cumulative default probability function



Source: Gregory (2012).

Figure 8 shows the cumulative default probability function. Now, the marginal default probability between t_1 and t_2 is given by:

$$q\left(t_1,\ \mathrm{t}_2\right) = F\left(t_2\right) - F\left(t_1\right) \quad \left(t_1 \le t_2\right)$$

And, the instantaneous default probability is given by the derivative of $F(t)$

1.9. Real world vs risk-neutral default probabilities

Real world default probabilities aim to reflect the true value of a financial underlying and correspond to the actual assessment of a counterparty default probability, and they are usually extracted from historical data and past default experience from similar counterparties. As a result, real world probabilities are useful for risk management purposes.

On the other hand, risk-neutral probabilities are derived directly from the market. The usual starting point would be the real-world probabilities and then several market implied parameters are added on such as *default risk premium*, and *liquidity premium*. As a result, risk-neutral probabilities reflect the market price and therefore are relevant for trading and hedging purposes.

Gregory argues that there is not conflict between these two default probability approaches, the difference relies exclusively in what they represent. (Gregory, 2012)

1.10. Recovery rates

Gregory defines recovery rates as the amount that would be recovered in the event of counterparty default. Recovery rates are usually expressed via *loss given default (*LGD*)*. (Gregory, 2012)

As recovery rates represent a portion (in percentage terms) of the exposure that would be recovered, in terms of loss (LGD) would be defined as $(1-R)$.

1.11. Collateral

Collateralization is one of the key elements when discussing counterparty credit risk. Collateral agreements are used to limit exposure in the event of a counterparty default. In contrast to the use of physical assets as security for debts, collaterals are under control of the counterparty in cash or securities, and therefore represent an instantaneous recovery in the event of default.

The amount of collateral is calculated based on the exposure or MtM of an open position. Therefore, according to the side of the exposure Party A could be asked to post collateral to Party B and if the exposure changes throughout the position lifetime collateral might be returned.

1.12. Foreign Exchange Options pricing

Option derivatives give buyers the right to buy or sell an underlying asset at an agreed price during a certain period of time. In the case of FX options, the underlying assets are always currencies. The buyer of an Fx option gets the right to buy or sell any currency, the foreign currency, at an exchange rate usually called the Strike price or FX forward price during a certain period of time. Please note that when we refer to "during a certain period of time" we are considering American type options, which enable the buyer to use his right at any moment during the option life. However, Colombian Fx Option rates are mainly European type, which gives the buyer the right to buy or sell only on maturity date.

As opposed to Stock Options pricing BS model, the Fx options pricing model needs to encompass two relevant interest rates which are also stochastic. As a result, it is important to describe a model that best suits the pricing of the most common types of options traded by financial institutions, European Fx options.

In 1983 Orlin Grabbe described a very accurate model for pricing foreign exchange options. In his paper *The pricing of call and put options on foreign exchange* published in the *Journal of International Money and Finance,* Orlin is able to derive close formulas for the pricing of Eurpean Fx options, starting from the very basics of interest rates economic equilibrium and Interest Parity Theorem, and finally moving forward through diffusion processes and Ito's Lemma (Orlin Grabbe, 1983).

According to Orlin, the pricing formula for European Call Fx Options is:

$$c(t) = S(t)B^*(t,\ T)N(d_1) - XB(t,\ T)N(d_2)$$

Where:

$S(t)$: is the spot domestic currency price of a unit of foreign exchange at time t.
$B^*(t, T)$: is the foreign currency price of a pure discount bond which pays one unit of foreign exchange at time $t+T$.
$N(d)$: is the standard normal distribution.
X: is the domestic currency exercise price of an option on foreign currency (Strike price).
$B(t, T)$: is the domestic currency price of a pure discount bond which pays one unit of domestic currency at time $t+T$.

And

$$d_1 = \frac{\ln\left(\frac{SB^*}{XB}\right) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{SB^*}{XB}\right) - \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$$

This very same model can also be written as:

$$c(t) = S(t)e^{-r_q(T_d - T_s)}N(d_1) - Xe^{-r_b(T_d - T_s)}N(d_2)$$

Where:

$S(t)$: is the Fx spot rate.
X: is the Strike price.
T_s: is spot date.
T_d: is the option delivery date.
r_b: is base currency interest rate for period (T_s, T_d).
r_q: is quote currency interest rate for period (T_s, T_d).

This model is also known as the Garman and Kolhagen BS extension for Fx Options.

2. CVA Literature review

Counterparty credit risk has been around for centuries, however, *Credit Value Adjustment* (CVA) is a relatively new introduced concept that emerged right after the subprime crisis.

CVA comes from a much larger subject of study named Counterparty Risk. In their book "Asset/Liability Management of Financial Institutions", Canabarro and Duffie define counterparty risk as the risk that Party A to an OTC fails to fulfil its contractual obligations causing losses to Party B (Canabarro & Duffie, 2003). As opposed to credit risk, counterparty risk is bilateral, it means that both parties face exposure according to the position they hold against each other.

Both, counterparty risk and CVA are based on the same concepts and it is difficult to explain the former without digging into the latter. However, CVA holds a specific purpose, and therefore its study has been intensified lately.

Simply put, one can describe CVA as the adjustment to any portfolio price by the integration of the counterparty's default probability. Currently, most of the pricing models are based in the assumption that we are trading against risk free counterparties, in fact, most of the discount interest rates used are also risk free. Nevertheless, the probability of the counterparty going bankrupt and, as a result, being unable to fulfil its derivative payments is far from cero *(0),* and needs to be quantified.

A different definition of CVA can be the difference between a risk-free priced derivative and its risk value which takes into account the counterparty's default probability. CVA can also be considered the cost of hedging from the counterparty's default probability.

Damiano Brigo, in his book Counterparty credit risk, collateral and funding, has also given a definition for CVA. He defines CVA as the difference between the value of a trade or a position traded with a default-free counterparty, it is said that the US Treasury is a default-free counterparty, and the value of this very same position traded with any counterparty (Brigo, 2013).

CVA can be defined in several ways, but all of them pursue the same objective described above. For instance, Jon Gregory defines CVA as the adjustment achieved by putting a value on the counterparty risk faced by an institution. According to this, the final price of a derivative is no longer the risky price calculated based on a risk-free assumption, but a final component must be added (In fact is subtracted) to correct for counterparty risk (Gregory, 2012).

Having mentioned these particular two approaches to what CVA is, Gregory defines the "real" value of a set of derivative positions in a simple formula.

$$Risky\ Value = risk\ free\ value - CVA \tag{1}$$

Consequently, according to Gregory, unilateral CVA as a standard formula is derived as follows:

$$Risky\ Value = \tilde{V}(t, T)$$

Where:

T: Maximum maturity date.
τ: Default time of the counterparty.

Consider the following scenarios:

2.1. Counterparty does not default before *T*

In this scenario, an indictor function is defined.

$$I(\tau > T)V(t, T) \tag{2}$$

Where $I(\tau > T)$ takes the value of 1 if default does not occur before T and zero otherwise

2.2. Counterparty does default before *T*

According to Gregory's derivation of CVA, we have to consider the payoff in two terms:

A. Cashflows paid up to the default time, which is basically the first scenario up to τ instead of T

$$I(\tau \leq T)V(t, \tau) \tag{3}$$

B. Default payoff
In this case, if Mark to Market, from now denoted as MtM, of the trade at default time $V(\tau, T)$ is positive then the institution will receive a recovery fraction *(R)* of the open derivatives positions, whereas if negative they will still have to pay the risk free amount. The latter will be affected by DVA later on, which is an institution's own credit risk.

$$I(\tau \leq T)\left(RV(\tau, T)^{+} + V(\tau, T)^{-}\right) \tag{4}$$

Where $x^{-} = \min(x, 0)$ and $y^{+} = \max(y, 0)$

Now, merging both payoffs, we have the following expression for the value of the position under risk-neutral measure:

$$\tilde{V}(t, T) = V(t, T) - E_Q\ (1 - R)I(\tau \leq T)V(t, T)^{+} \tag{5}$$

As the expression above expresses the value of the portfolio adjusted by the counterparty risk, CVA by itself will be described as:

$$CVA(t, T) = E^{Q}\ (1 - R) I(\tau \leq T) V(\tau, T)^{+} \tag{6}$$

Please note that according to the expression above, CVA counts only for the positive MtM positions by the expression $V(\tau, T)^{+}$ as these are the ones an institution is exposed to if counterparty default occurs before T.

Even though the equation seems to have little to no complexity, the fact is that it is not linear due to the incorporation of risk mitigants such as netting and collateral.

As a result, Gregory defines CVA in a probability approach as follows:

$$CVA \approx (1 - \text{Rec}) \sum_{i-1}^{m} DF(t_i) EE(t_i) PD(t_{i-1}, t_i) \tag{7}$$

Where:

- $(1 - Rec)$ *Loss given default* or LGD. In the event of a counterparty default, some percentage of the amount due would be recovered, thus this expression defines the amount that would be lost.
- (DF) *Discount Factor*. This is the risk-free factor that will be used to discount future payoffs.
- (EE) *Expected exposure*. This is the expected value to be lost in the event that the counterparty defaults.
- (PD) *Default probability*. This expression describes the marginal default probability in the interval t_{i-1} and t_i.

Even though Gregory and Brigo have stated a formula for CVA as presented above, the CVA model may suffer variations depending on the type of product that is being priced. For instance Hui Li presented a CVA approach for Credit Default Swaps in 2008 (Li, 2008).

In order to achieve the CVA calculation for these types of trades, Hui Li makes some assumptions. First, counterparty risk will be unilateral on the protection of the seller, this due to the fact that most of the CDS negative basis trades were traded at low spreads that lead to a negative MtM. Second, credit quality of the counterparty is independent of the Collateral performance. Finally, Li assumes a recovery rate R as constant in the level of 40%, which according to Basel III, is the market practice (Li, 2008; Reynolds *et al*., 2013).

As a result of the assumptions mentioned above, the CVA for CDS can be expressed as:

$$CVA = (1-R)\int_0^T EE(t)\,dPD(0,\ t) \tag{8}$$

Where $EE(t)$ expresses the risk-neutral discounted expected exposure, and $PD(0, t)$ expresses the risk-neutral probability of counterparty default between time 0 and t.

Now Hui Li defines another assumption. Considering that $EE(t)$ is the sum of risk-neutral discounted expected payoffs at or after t under a CDS contract, one could define an expression $-dEE(t)$ as the risk-neutral discounted values of expected cash flow at t. Then CVA is re-expressed as:

$$\begin{aligned} CVA &= (1-R)EE(t)PD(0,\ t)\,|_0^T - (1-R)\int_0^T PD(0,\ t)\,dEE(t) \\ CVA &= (1-R)EE(t)PD(0,\ t)\,|_0^T - (1-R)\int_0^T PD(0,\ t)\,dEE(t) \\ &= (1-R)\int_0^T PD(0,\ t)\big(-dEE(t)\big) \\ &= (1-R)\int_0^T \big(1-PS(0,\ t)\big)\big(-dEE(t)\big) \\ CVA &= (1-R)EE(0)-(1-R)\int_0^T \big(PS(0,\ t)\big)\big(-dEE(t)\big) \end{aligned} \tag{9}$$

$$CVA = (1-R)EE(0)-(1-R)\int_0^T \big(PS(0,\ t)\big)\big(-dEE(t)\big) \tag{10}$$

According to Hui Li, this can be done when considering $PS(0, t)$ as the survival probability between time 0 and t. And $EE(0)$ is the current value of the ABS CDO insurance portfolio with no counterparty risk (Li, 2008).

In his paper, Hui Li separates even further the expressions of CVA detailed above and comes up with a semi analytical expression of CVA for CDS on super senior ABS CDO. Even though, these types of products are not traded in Colombia, it is important to understand that some variations of the CVA formula depicted earlier need to be done in order to simplify calculations according to the specificities of each product.

Another important work on CVA calculation was done by Lu Dongsheng and Juan Frank, in which they presented a more efficient CVA calculation methodology based on a backward framework under risk-neutral probabilities (Lu & Juan, 2010).

The model presented by Lu and Juan is the following:

$$CVA = E^{Q}\left[\left(1-R\right)D_{t}\min\left(Threshold,\ \mathrm{V}_{t}\right)_{|\,A<H_{d}}\right] \tag{11}$$

Where A is the trade value, H_d is the default threshold, R represents the recovery rate, D represents the discount factor and Q expresses risk-neutral measure. In this context, the CVA value is the result of the integration of all future exposures discounted to time 0. In that case, the implied formulation is only concerned about those scenarios in which the counterparty defaults.

In line with the model presented above, the process would involve the following steps:

a) Scenario generation.
b) Valuation under market generated scenarios.
c) Collect CVA by aggregating defaulted exposures.

Lu Dongsheng and Juan Frank suggest that the model presented has several advantages compared to other CVA approaches. Firstly, the model is efficient and flexible in terms of computational requirements. This is achieved by eliminating redundant calculations and focussing on incremental CVA. Secondly, due to the large number of simulations, the model is more accurate and less impacted by noise. Finally, the model is highly customizable and can be adjusted and calibrated easily (Lu & Juan, 2010).

Moreover, in 2012 John Hull and Alan White suggested a model in which wrong way risk was incorporated into the CVA calculation using Monte Carlo Simulation. According to their paper, Wrong Way risk, which is the positive correlation between an institution's exposure and its counterparty's default probability, can be incorporated into CVA assuming a relationship between the hazard rate and the value of variables generated as part of Monte Carlo simulations (Hull & White, 2012).

Hull and White define CVA as follows:

$$CVA = \left(1 - R\right) \int_{t=0}^{T} q\left(t\right) v\left(t\right) dt \tag{12}$$

where T is the longest derivative maturity date, $v(t)$ is the derivative's payoff at time t, R is the Recovery rate, and $q(t)$ is the probability density function to counterparty default under risk-neutral measure.

Now, using Monte Carlo simulation, the integral in the equation above can be approximated as:

$$CVA = \left(1 - R\right) \sum_{i=1}^{n} q_i v_i \tag{13}$$

where q_i is the default probability between times t_{i-1} and t_i, this variable is usually calculated from credit spreads, CDS are a good measure for counterparty's default probabilities, but, when trading with real sector counterparties, similar company credit spreads might be useful.

Now, in order to incorporate wrong-way risk into CVA calculation, Hull and White consider the introduction of a model in which $q(t)$ depends on the evolution of the variables simulated through Monte Carlo up until time t. Consequently, a hazard rate h that measures the probability of a default occurrence within any short period of time Δt conditional on no earlier default is introduced into the model.

As h depends on the behavior of different variables x, h can be expressed as follows:

$$h(t) = f(x(t)), \tag{14}$$

where f is a function that has the property that $h(t) \geq 0$ for all possible values of $x(t)$.

Hull and White argue that the model proposed can be used to incorporate credit triggers, this means that a relationship between the counterparty's credit spreads and its credit rating can be introduced into the CVA model.

Finally, in order to illustrate the nature of wrong-way risk using the function above, Hull and White consider the following approach:

$$h(t) = \exp\left[a(t) + bw(t)\right] \tag{15}$$

where b is a constant parameter that measures the amount of wrong-way risk in the model, $a(t)$ is a function of time and w is the value of a derivative's portfolio (Hull & White, 2012).

Another interesting approach of CVA is the one presented by Sidita Zhabjaku in 2013. In it, Sidita introduces a model for CVA using defaultable options that aims at relating the company's instantaneous rate of default to its stock price. The author argues that this approach is valid since it relies on the fact that there is always a plunge in the stock before any default event. In order to accomplish this, Sidita presets CVA as the difference between a non-defaultable option and a defaultable option price (Zhabjaku, 2013).

Sidita describes a preliminary concept of CVA previously presented by Mats Kjaer in 2011 as follows:

$$\breve{V}(t)=V(t)-\psi(t) \tag{16}$$

where $V(t)$ is the value of a risk-free portfolio, $\breve{V}(t)$ is the value of a portfolio in which default can occur and $\psi(t)$ represents CVA (Kjaer, 2011).

Reexpressing the equation above we have unilateral CVA as:

$$CVA = (1 - R_C)\int_t^T \mathrm{E}\left[V^+(u)\frac{N(t)}{N(u)} | F_t\right] f_\tau(u)du \tag{17}$$

Where $\mathrm{E}\left[V^+(u)\frac{N(t)}{N(u)} | F_t\right]$ is the expected exposure as $V^+(u) = \max(V(u), 0)$ and $f_\tau(u)$ is the density of default time τ.

As default time is exponentially distributed, then τ can be simulated through the inverse cumulative distribution of an exponential as follows:

$$P(\tau \leq T) = 1 - e^{-\lambda\tau} \tag{18}$$

As a result, Sidita expresses an analytical form of defaultable option with exponential default arrival times driven by λ, and finally merged with Black-Scholes formula for call options:

$$R\int_0^T C_{BS}(\tau)P_\lambda(\tau)d\tau + C_{BS}(T)\int_{\tau=T}^{\infty} P_\lambda(\tau)d\tau \tag{19}$$

Where $P_\lambda(\tau) = \lambda e^{-\lambda\tau}$ and $C_{BS}(T)$ is the Black-Scholes formula for a call option under a risk-neutral measure. Then, Sidita continues elaborating on the incorporation of the stock price within the defaultable option and resulting in two methods later tested (Zhabjaku, 2013).

3. Models to be discussed

It is time to present the models to be discussed in this paper. The first model to be described is the discounted cash flow approach model for CVA, which is usually implemented by Colombian financial institutions due to its lack of computational needs and low implementations costs.

3.1. Model 1: CVA Discounted cash flows approach

Some financial institutions in Colombia have decided to apply simpler models for unilateral CVA than the theoretical and more robust models in order to meet the regulator requirements. However, as explained earlier, because of the lack of robust systems and calculation engines as well as budgetary constraints, the current models applied by these institutions may lack some important components from theoretical models. The following model is also described as the *Discounted cash flow approach* and is used by most financial institutions in Colombia.

As the current value of any derivative is calculated by discounting future cash flows using *risk-free* interest rates, the discounted cash flow approach for unilateral CVA will involve an additional credit spread to the *risk-free* rate in order to represent counterparty credit risk.

Discounted cash flow approach can be presented as:

$$CVA = FV_{Rf} - FV_{Credit\ adjusted} \tag{20}$$

where:

FV_{Rf}: is the risk free fair value calculated by any conventional pricing model.
$FV_{Credit\ adjusted}$: is the fair value calculated by using credit spreads on top of the risk free rates for discounting future cash flows.

3.1.1. FX Options CVA Discounted Cash Flows Approach

In the case of Fx Options, as pricing is done through the Black-Scholes model the CVA adjustment is developed from the APT (Arbitrage Pricing Theory) model which is based on *Expected Loss* (EL). As a result, CVA for Fx Options is defined as:

$$CVA = BS * (1 - EL_a) \tag{21}$$

where:

BS: is the Fx Option price as an outcome of the Black Scholes model.
EL_a: is the *Expected Loss adjustment* and is based on default probability and *Loss Given Default* (LGD).

$$EL_a = (DP) * LGD \tag{22}$$

Where *loss given default (*LGD*)* is assumed to be 60% of the outstanding amount according to Basel III. *Default probabilities* are calculated according to an internal credit risk model and for thesis purposes it is assumed as given.

As it is possible to evidence, the current model does not account for *expected exposure* and it requires the construction and modelling of reliable credit spread curves for each and every single counterparty. Montecarlo simulation is also not part of the process, as it requires a high computational processing to run over a spreadsheet. However, the model is simple and easy to implement, also it does not require sophisticated pricing engines and can also be applied on a transactional level. For the reasons explained above, the model is one of the best choices for financial institutions around the world and meets the minimum requirements for CVA accountability.

3.2. Model 2: Unilateral CVA Theoretical model

The Theoretical CVA model as discussed earlier is relevant as it encompasses important variables such as *Potential Future Exposure* and *Expected Exposure*. In addition, this CVA approach integrates over time in order to take into account the distribution of EE and PD, and when applying Montecarlo simulations, it is possible to determine the cost of CVA for n scenarios.

As mentioned before and based on Gregory's simple CVA approach explained we have two cases to consider starting from the *Risky Value* $\tilde{V}(t, T)$; Counterparty does not default before T described in formula (2) and Counterparty does default before T.

As the second case consists of two terms, *Cashflows paid up to the default time* formula (3) and *Default payoff* formula (4), it is necessary to put all payoffs together as follows:

$$\tilde{V}(t,\ T) = E^{Q}\Big[I(\tau > T)V(t,\ T) + I(\tau \leq T)V(t,\ \tau) + I(\tau \leq T) \\ \left(RV(\tau,\ T)^{+} + V(\tau,\ T)^{-}\right)\Big] \tag{23}$$

Remember that $V(\tau, T)^{+} = \max(V(\tau, T), 0)$ and using $x^{-} = x - x^{+}$ and rearranging we have:

$$\tilde{V}(t,\ T) = E^{Q}\Big[I(\tau > T)V(t,\ T) + I(\tau \leq T)V(\tau,\ T) + I(\tau \leq T) \\ \left((R-1)V(\tau,\ T)^{+} V(\tau,\ T)\right)\Big]$$

Now, considering that $V(t, \tau) + V(\tau, T) = V(t, T)$ we have:

$$\tilde{V}(t,\ T) = E^{Q}\Big[I(\tau > T)V(t,\ T) + I(\tau \leq T)V(t,\ T) + I(\tau \leq T)\left((R-1)V(\tau,\ T)^{+}\right)\Big]$$

Finally, since $I(\tau > T)\, V(t,\ T) + I(\tau \leq T)\, V(t,\ T) = V(t,\ T)$ and rearranging, we have the following equation for the *Risky Value*:

$$\tilde{V}(t, T) = V(t, T) - E^{Q}\left[(1-R)I(\tau \leq T)V(\tau, T)^{+}\right] \tag{24}$$

Where:

$$CVA(t, T) = E^{Q}\left[(1-R)I(\tau \leq T)V(\tau, T)^{+}\right]$$

Now, as it is very important to determine the exposure at the default date τ we need to integrate over all times before the final maturity date:

$$CVA(t, T) = (1-R)E^{Q}\left[\int_{t}^{T} B(t, \tau)V(\tau, T)^{+} dF(t, \tau)\right]$$

Where:

$B(t, \tau)$: is the risk-free discount factor
d$F(t, \tau)$: is the instantaneous default probability as $F(t, \tau)$ is the cumulative default probability for the counterparty.
As the portfolio's positive values are represented by the *Expected Exposure (*EE*)*: we have $EE(\tau, T) = E^{Q}\left[V(\tau, T)^{+}\right]$.

Finally, we can rewrite the expression as follows:

$$CVA(t, T) = (1-R)\left[\int_{t}^{T} EE(\tau, T)B(t, \tau)dF(t, \tau)\right] \tag{25}$$

For the purpose of this thesis, the term $EE(\tau, T) = E^{Q}\left[V(\tau, T)^{+}\right]$ will be calculated using formula (28) which is the value for a European Call Fx Option.

Please also note that $(1-R)$ represents Loss Given Default, which for the purpose of model 2, is the constant given. It is worth mentioning that according to article 161 from the Capital Requirements Regulation, institutions may use an LGD of 45% for senior exposures without eligible collateral or for senior purchased corporate receivables exposures where an institution is not able to calculate PDs or the estimates do not meet the requirements set out in section 6 from the same document (Council, 2013).

3.2.1. Options Theoretical CVA Approach

When dealing with long Option positions a simplification for CVA can be done since the Expected value can never be negative. According to Gregory, CVA for a long Option position can be expressed as follows:

$$CVA_{option}(t, T) = (1-R)E^{Q}[I(\tau \leq T)]E^{Q}[B(t, \tau)V_{option}(\tau, T)]$$

$$CVA_{option}(t, T) = (1-R)F(t, T)V_{option}(t, T) \tag{26}$$

Where V_{option} is the premium value usually paid upfront.

3.3. Model 3: Unilateral CVA Theoretical model with non-constant LGD

The third model discussed in this thesis paper is closely related to the unilateral CVA theoretical approach explained in model 2, but with a non-constant LGD. Although, theoretical CVA models when applied properly are much more reliable and accurate compared to the simpler CVA version from model 1, it is also important to consider that LGD should not be taken lightly. LGD's impact on CVA calculation could not be underestimated, therefore it is important to consider a model in which LGD is not a constant given.

Stefano Bonini and Giuliani Caivano in their paper *Econometric approach for Basel* III *Loss Given Default Estimation: from discount rate to final multivariate model* state a simple LGD workout model that can be easily merged in our theoretical CVA model.

The *Workout* LGD *calculation* model is based on the economic notion of including all relevant costs that are implied in the recovery process represented by R. This model covers all guidelines related to discounted cash flows from the Committee of European banking supervisors (Bonini & Caivano, 2017).

According to Bonini and Caivano the Workout LGD can be expressed as follows:

$$LGD = (1 - R) = 1 - \frac{\sum \mathrm{Rec}_i \delta_i^T - \sum A_i \delta_i^T - \sum Cost_i \delta_i^T}{EAD} \tag{27}$$

Where:

Rec_i: represents the recovery flow at date i.
A_i: is the increase flow at date i.
$Cost_i$: represents the cost of litigation, collection procedures, legal expenses at date i.
i: is default date.
EAD: Exposure at default.
δ_i^T: is the discount rate of each flow at date i.

4. Model construction and assumptions

The models were set up in MATLAB and run for a long single Call FX Option with the following parameters:

Notional	USD 1.00
Days to maturity	90 – 1800
Spot price	$ 3,100.00
Strike price	$ 3,200.00
Market implied volatily	10.50%
Recovery rate	40%
USD and Cop Interes rates	Precia

4.1. Model set up

Models 2 and 3 are based on Monte Carlo simulation built in matlab code divided into 5 stages: I recommend enumerating the stages as a list.

4.1.1. Market data loading and construction

The first step is to upload market data related to foreign and local interest rates. The information loaded to the models is built by *Precia* and is described in their pricing paper where USDCOP FX derivatives must be discounted using USDIBR implied foreign interest rates and COPIBR implied local interest rates. IBR is the local interbank funding rate.

The files used are:

- SwapCC_IBR_Nodos *(Local interest rate curve)*.
- Tasas_USDIBR_Nodos *(Foreign interest rate curve)*.

Both of the files are built in a term structure instead of daily based rate curves.

These curves are loaded to the models and saved in *rb (local)* and *ra (foreign)* vectors which later are referred for each step calculation using linear interpolation.

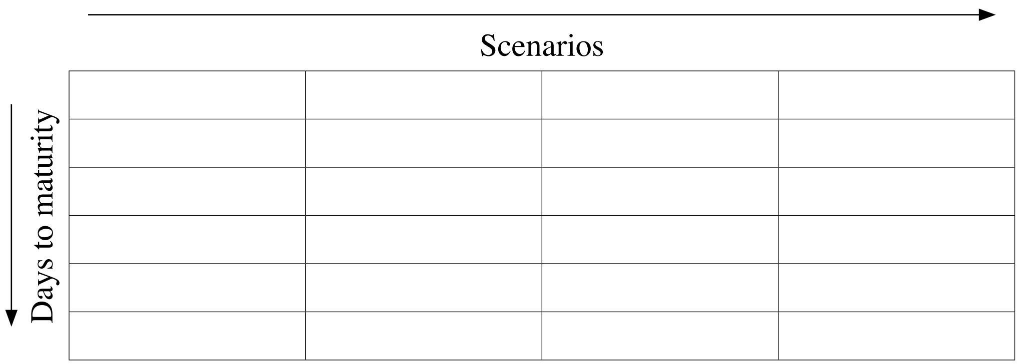
In addition, a term structured set of Hazard rates *h* is loaded by each counterparty and saved in a vector for survival and default curves construction. This curve must be provided as an input, and for the purpose of this thesis paper, we will assume it given as follows:

Term	h Rate
0	0
360	4.94%
720	6.67%
1080	8.54%
1440	10.60%
1800	12.95%

Other market data values (Spot rate, Market implied vol, days to maturity) are set by the user according to market values of the desired date.

4.1.2. Spot rate matrix

In order to run Monte Carlo simulation, a set of Spot prices is needed which later will be used as the starting point for forward rates calculation at each step. This matrix is called *S01* and is built following a Geometric Brownian motion. All Matrixes are built as follows:



Values for *S01* matrix are calculated using the following expression:

$$S01_{i,j} = S(0)_{i-1,j}\, e^{\left((r_{b1}-r_{a1})*\frac{1}{365}-\frac{1}{2}*\sigma^2*\frac{1}{365}+\sigma*\sqrt{\frac{1}{365}}*dwt\right)}$$

Where:

r_{b1} and r_{a1}: are the local and foreign interest rates for 1 day term.
$S(0)_{i-1,j}$: is the previous step USDCOP spot rate.
σ: is the USDCOP volatility.
dwt: is the stochastic term.

As a consequence, the matrix is the result of a spot projected one step at a time to the maturity of the contract.

4.1.3. Fwd rate matrix

The forward rate matrix *(*ST*)* is calculated based on the spot rate matrix previously explained. In this matrix, each value is computed starting from the corresponding spot rate at each step and then calculating a forward rate to the maturity of the contract using the following expression:

$$ST_{i,j} = S(0)_{i,j}\, e^{\left((r_{b_{d-i}}-r_{a_{d-i}})*\frac{d-i}{365}-\frac{1}{2}*\sigma^2\frac{d-i}{365}+\sigma*\sqrt{\frac{d-i}{365}}*dwt\right)} \Big| i:d$$

Where:

d: is days to maturity and $d-1$ is the remaining days to maturity as each step moves forward.
$S(0)_{i,j}$: is the spot rate at step i for each scenario.
$r_{b_{d-i}}$ and $r_{a_{d-i}}$: are the local and foreign interest rates for term $d-1$ calculated from each curve structures by linear interpolation.

4.1.4. Payoff matrix

The payoff matrix *(Payoffm2)* is the result of taking each forward rate and calculating the discounted FX Call Option payoff at each step using the following expression:

$$Payof\ fm2_{i,j} = \max\left(ST_{i,j} - K, 0\right) * e^{\left(-r_{b_{d-i}}\right) * \frac{d-i}{365}}$$

Where:

$ST_{i,j}$: is the projected forward rate for the remaing days to maturity at each step.
K: is the agreed stroke Price of the FX Call Option.

4.1.5. EE vector

The Expected Exposure vector *(EE)* is the result of adding up the positive values for each step and dividing it by the number of scenarios of the simulation. For derivatives different from options, where payoffs can be negative, it is important to consider only positive values as they represent the exposure from a counterparty default and therefore CVA calculation is needed. As the purchase of an FX Call Option limits the losses to zero by itself, this vector is simply the result of the average of the payoff values.

$$EE_{i,1} = \text{mean}\left(Payof f_{i:nrep}\right)$$

The EE vector is an ix1 size as shown below:

	1
Days to maturity ↓	

4.1.6. Cumulative survival probability curve

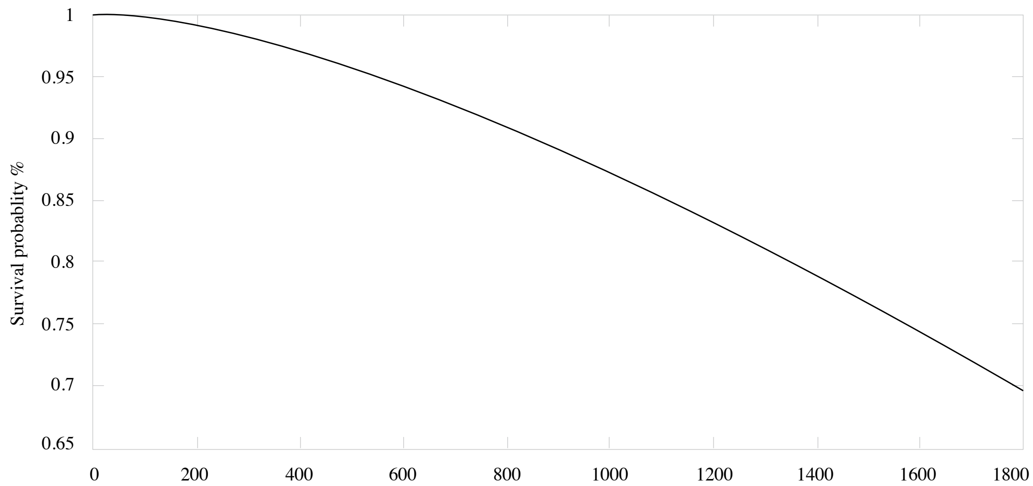
The cumulative survival curve *(*SU*)* is calculated for all steps of the simulation using the following expression:

$$SU_{i,1} = SU_{i-1,1} * e^{(-h)*\frac{1}{365}}$$

Where:

$SU_{1,1} = 1$, since the default probability for a counterparty at day 1 is 0.
h: is the hazard rate, which represents the intensity of default. This value is dependent on each counterparty and for the purpose of this thesis is assumed a constant given.

Figure 9: Cumulative survival probability

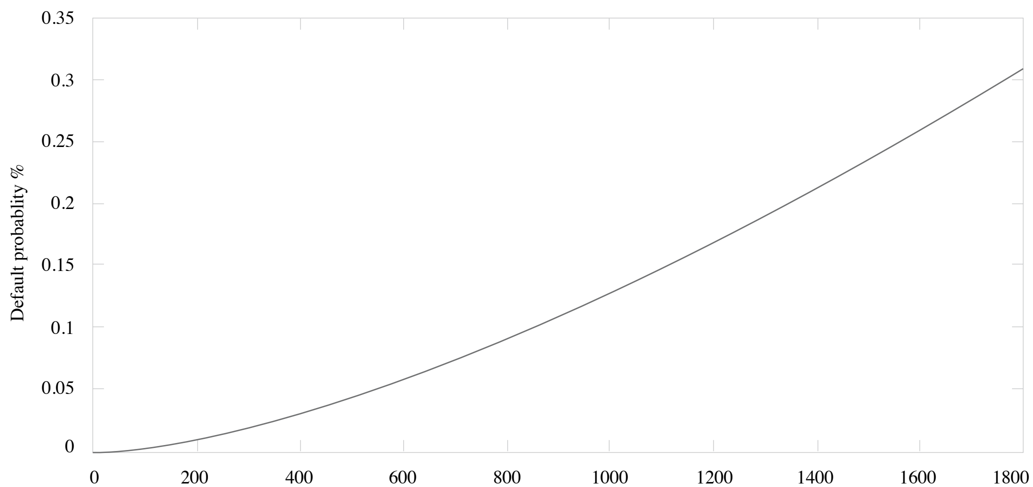


4.1.7. Cumulative default probability

The cumulative default probability curve *(F)* is calculated based on the Cumulative survival probability, and is given by the following expression:

$$F_{i,1} = 1 - SU_{i,1}$$

Figure 10: Cumulative default probability

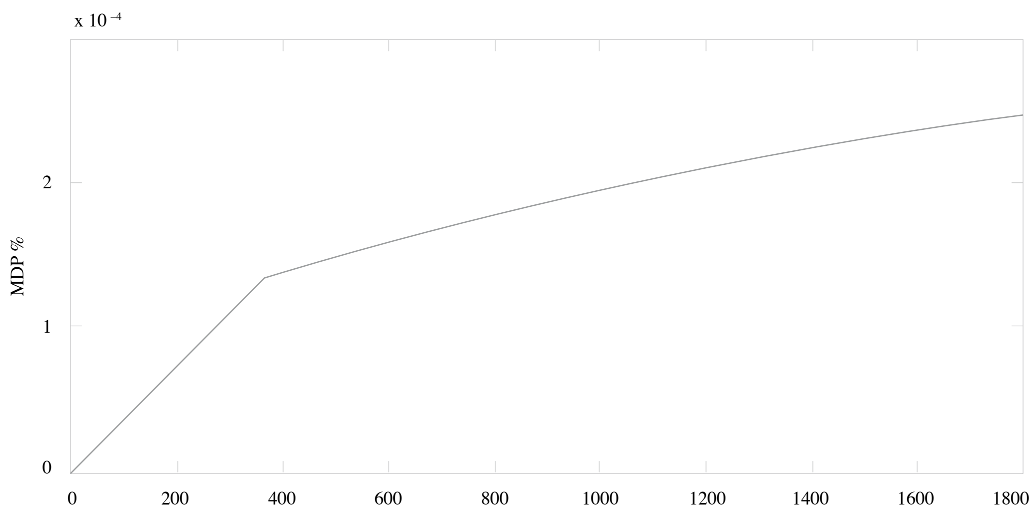


4.1.8. Marginal default probability

The marginal default probability (MDP) is a vector that represents the marginal increase of default probability at each step of the simulation. This calculation is crucial to calculate the expected default values. This vector is computed following the expression:

$$MDP_{i,1} = F_{I,1} - F_{i-1,1}$$

Figure 11: Marginal default probability

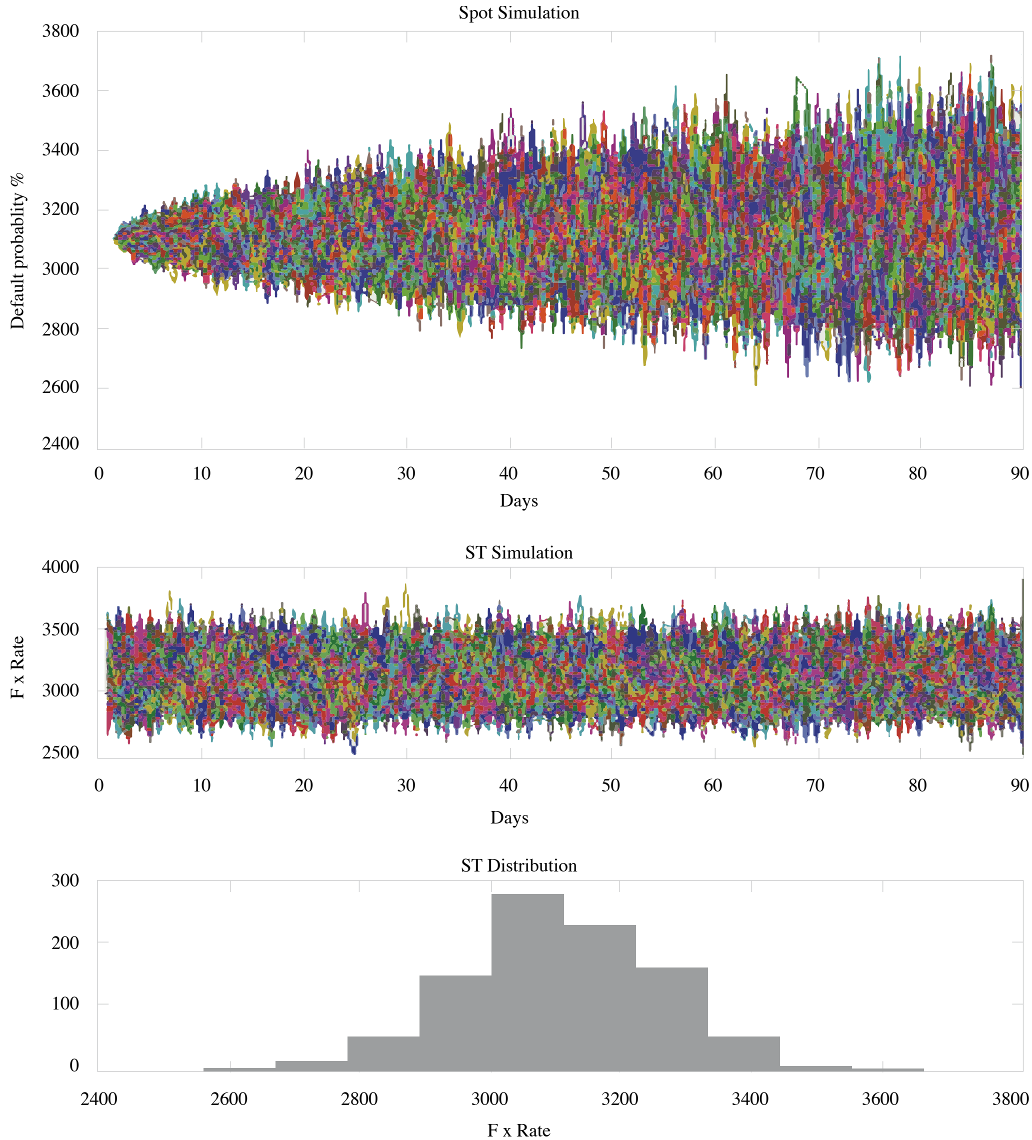


5. Results and conclusions

The model was run on different maturity dates for a single FX Call Option following the assumptions described in this document and according to the financial information shown above.

Both, the FX Spot and FX Forward in the following figures follow a Geometric Brownian motion normally distributed over a 90-day term:

Figure 12: Simulations



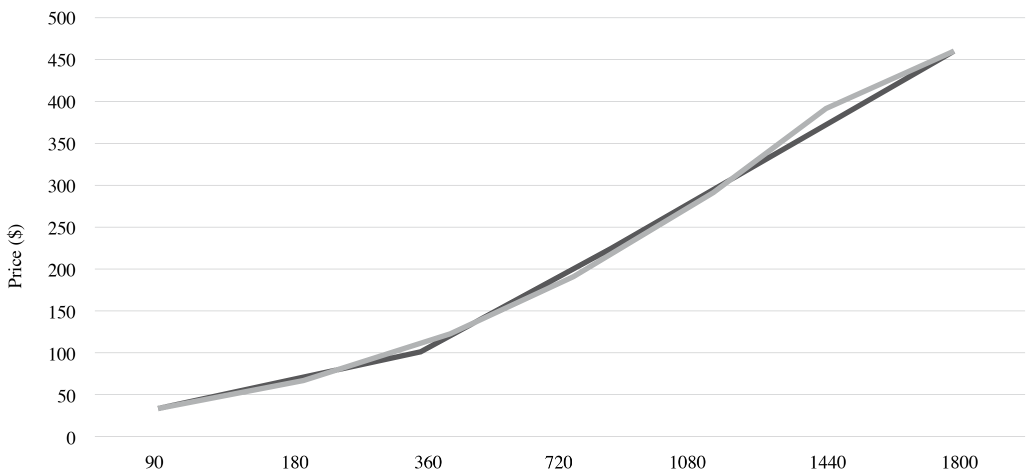
Once the scenarios were simulated for the first term date, we proceed to calculate the Fx option Value for all 3 models and their respective CVA and are displayed in the following table 1:

Table 1: Models results

	Model 1 (Non Simulated)			Model 1 (Montecarlo)			Model 1 (Workout LGD)			
TERM	Call Price BS	CVA1	CVA Fair Value 1	Call Price MC	CVA2	CVA Fair Value 2	CVA3	CVA Fair Value 3	Mean EE	MPFE
90	28.2052	0.0255	28.1797	28.3051	0.0259	28.2792	0.0302	28.2749	28.4068	34.0852
180	55.5478	0.2014	55.3464	62.5381	0.2058	62.3323	0.2401	62.298	56.3318	66.3945
360	101.377	1.4612	99.9158	107.8605	1.4777	106.3828	1.724	106.1365	101.8152	120.4625
720	184.6124	8.6782	175.9342	178.4454	8.7692	169.6762	10.2308	168.2146	184.6844	223.9944
1080	272.2125	23.6707	248.5418	270.1998	23.639	246.5608	27.5789	242.6209	268.693	341.8451
1440	364.9665	48.5896	316.3769	385.1043	48.851	336.2533	56.9929	328.1114	363.9067	483.004
1800	459.6403	84.708	374.9323	460.3837	86.9374	373.4463	101.427	358.9567	461.2913	663.7549

First of all, it is important to point out that Fx Option prices differ from one BS model to the Monte Carlo simulated one. Besides choosing from 1000 scenarios, there are important differences to consider when taking into account that this is a 1USD notional derivative. Figure 8 shows that the MC simulated model computes higher prices almost at all terms all the way to 5-year positions.

Figure 13: Fx Option Prices BS vs MonteCarlo



However, besides the differences in prices pointed out above, Figure 9 shows that CVA values are very close to all terms when comparing the CVA discounted Cash flows approach (Model 1) with the CVA Theoretical model (Model 2), which by definition, considers the Expected Exposure to all t_i steps on the simulation and the Marginal default Probabilities. Model 2 CVA is higher in almost all term simulations and could be important when considering different asset classes and real market open positions.

On the other hand, Figure 9 also shows that the values for CVA 3 which resulted from Model 3, which considers a LGD workout approach are considerably higher than 90-day terms and increases to higher levels than the other models as the maturity is further in the future.

It is important to point out that both Models 2 and 3 are accountable for measures such as PFE and Maximum PFE (MPFE) since these approaches consider scenario simulations. Measures such as PFE and MPFE are very important when pricing CVA and could become part of key analyses when considering counterparty default events. The following chart shows the Exposure profile of Models 2 and 3 for a 1-year term.

Figure 14: CVA models comparison

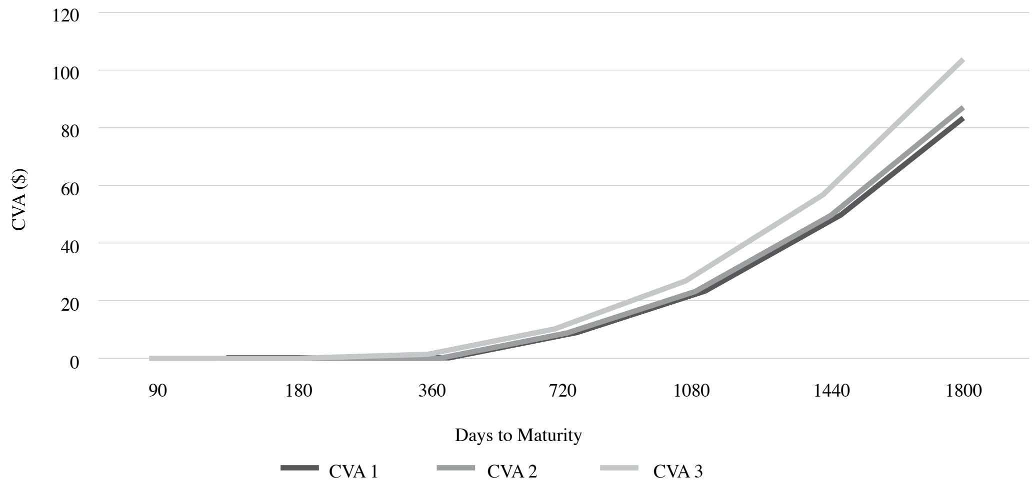
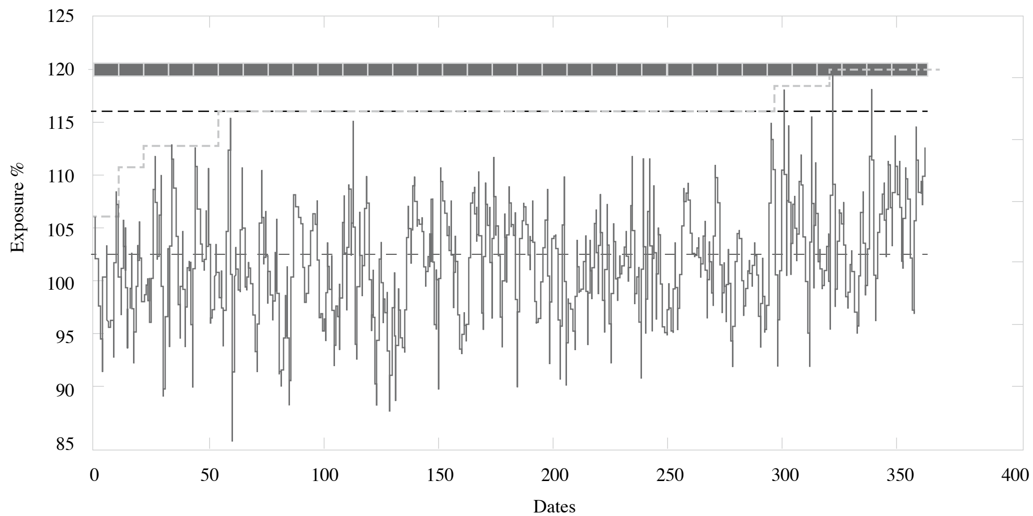


Figure 15: Exposure profile



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