

Optimal Early Termination in PPP Projects Based on Real Options Theory

**Terminación anticipada óptima en proyectos de
APP basada en la teoría de opciones reales**

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Abstract

Early termination is a contractual compensation mechanism implemented in PPP infrastructure projects where private investors have the right to abandon the project to mitigate risks such as traffic risk. In this paper we used the Real Options Theory to value early termination as an option to abandon a project where both the traffic risk and the compensation fee are modelled assuming correlated stochastic processes. The option pricing model was implemented with an analytical solution and results were compared with the Monte Carlo simulation technique. The results confirm the benefits that the contractual mechanism offers to private investors to mitigate risks, while at the same time improving a project's value.

Key words: Infrastructure projects; option to abandon; early termination mechanism.

JEL Clasificación: C14, G12, G13.

Resumen

La terminación anticipada es un mecanismo de compensación contractual implementado en proyectos de infraestructura de APP, donde los inversionistas privados tienen derecho a abandonar el proyecto para mitigar riesgos como el de demanda. En este artículo utilizamos la teoría de opciones reales para valorar la terminación anticipada como una opción de abandono de un proyecto, en el que el riesgo de demanda y la tarifa de compensación se modelan asumiendo procesos estocásticos correlacionados. Así, se implementa el modelo de valoración de opciones con una solución analítica y se comparan los resultados con la técnica de simulación Monte Carlo. Los resultados confirman los beneficios que el mecanismo contractual ofrece a los inversionistas privados para mitigar riesgos y mejorar el valor del proyecto.

Palabras clave: proyectos de infraestructura; opción de abandono; mecanismo de terminación anticipada.

Códigos JEL: C14, G12, G13.

Introduction

Public-private partnerships (PPPs) have been used to incorporate the private sector in infrastructure projects. Through the PPP scheme, governments can concede an infrastructure project to a private party under a concession agreement to finance, build and operate it for a long time (Yescombe, 2002;

Grimsey and Lewis 2002; Liu et al., 2015). Additionally, a special purpose vehicle (SPV)—i.e., the company responsible for the project, is created under an ad hoc base where cash flows are the primary method of loan repayment for lenders. However, PPP projects are exposed to several risks which must be managed properly. Therefore, an analysis of risk is essential in PPP projects as Gatti (2008) suggested.

Gatti (2008) argued that the division of risk between both public and private parties is necessary given that the SPV is exposed to losses when risks are not managed properly. Furthermore, when some risk events occur, PPP projects may be forced to terminate early. For example, according to reports published by the World Bank, 661 of 7,120 projects terminated earlier between 1980 and 2018. Similarly, Wang et al. (2004), Guasch et al. (2007), Caselli et al. (2009), Xiong et al. (2015), and Ameyaw & Chan (2015) also found that many PPP projects have suffered difficulties due to poor risk management and some of these have had to be terminated earlier.

Talus (2009) found some reasons for early termination, such as asymmetric information between private and public parties, poor risk management, changes in the law, events of force majeure, and others. Additionally, Guasch & Straub (2009), Galilea & Medda (2010) and Iossa & Martimort (2016) found that inexperience in government institutions, as well as corruption and untrustworthiness of public officials, are some reasons for the early termination of PPP projects. Furthermore, these problems generate failures in PPP contracts as Zhang & Xiong (2015) and Song et al. (2018) have explained.

Additionally, early termination results in even higher expenditure for the government due to the huge compensation owed to the concessionaire alongside the additional financial costs incurred. Therefore, there are still many challenges for PPPs, among which renegotiations and early terminations are two issues to be tackled. As Iossa & Martimort (2016), Giraldo (2019) and Xiong & Han (2021) have suggested, the main concern for government is identifying the best method to arrive at a value through the compensation mechanism.

Despite the vast literature on PPP projects that tackles risk analysis and valuation (Grimsey & Lewis, 2002; Iyer & Sagheer, 2011; Ashuri et al., 2012; Liu et al., 2014; Attarzadeh et al., 2017; Carbonara & Pellegrino, 2018), there is still a gap in the empirical literature to provide completely effective compensation mechanisms in PPP projects when early termination takes place. In that sense, we propose an option pricing approach to value a compensation mechanism in PPP projects based on a real option approach. To do so, we

value a termination mechanism as a real option to abandon by considering a deterministic and stochastic compensation fee. Additionally, both the traffic risk and the compensation fee are modelled assuming correlated stochastic processes where the termination fee involves the dynamics of the cash-flows and the main uncertainty sources. Finally, the model is implemented for a road infrastructure project with an analytical solution, and the results are compared with the Monte Carlo simulation technique.

This paper is organized as follows. Section 1 presents a literature review of the real options theory in PPP projects to value compensation mechanisms such as early termination. Section 2 presents the model set up to value this contractual mechanism. Finally, Section 3 presents a numerical example and Section 4 concludes.

1. Literature Review

Song et al. (2018) stated that early termination is a failure of the PPP contract caused by the parties or by force majeure events that impacts the construction or the operation phase of the project. According to Liu et al. (2017), Giraldo (2019) and Zapata and Mejia (2022), early termination in PPP projects is most frequently due to factors such as low profits, social pressure from end-users, and inaccurate market predictions. Therefore, not only is the contractual design of compensation mechanism for early termination important for both the public and private parties, but also the implementation of suitable pricing models to determine the corresponding value. In fact, the real options theory (ROT) arises as a useful tool to value compensation mechanisms regarding early termination.

The ROT arises as a useful tool for making optimal investment decisions and constitutes a better tool for assessing investment projects under uncertain conditions. To do that, ROT uses the financial option pricing models developed by Black & Scholes (1973) and Merton (1973) (Brennan & Schwartz, 1985; McDonald & Siegel, 1986; Dixit & Pindyck, 1994; Trigeorgis, 1996), which characterize the investment projects compared to the traditional methods such as NPV or IRR, which were typically used to value PPP projects as well as indemnification mechanisms. Additionally, ROT has found numerous applications in the field of infrastructure projects as Martins et al. (2015) and Zapata and Mejia (2022) have suggested. Several studies have been developed to value government compensation mechanisms including revenue guarantees as well as the design of optimal contracts through contractual flexibilities (optimal prices

or optimal time development) as we show above. However, the literature about the design of mechanisms to deal with the early termination of the concession contract has been poorly developed.

In the road infrastructure field, the government usually incorporates compensation mechanisms as contractual support to mitigate risks like the demand risk (Cheah & Liu, 2006; Brandão & Saraiva, 2008; Iyer & Sagheer, 2011). For example, if the traffic level is very low, the government can support the SPV to guarantee an expected rate of return or net present value. All the compensation mechanisms have been widely used based on the real options theory (ROT), through which it is necessary to use an adequate tool to model the dynamics of the underlying uncertainties and the pricing mechanisms (Martins et al., 2015; Attarzadeh et al., 2017). In fact, the ROT has found several applications in the infrastructure sector to value contractual guarantees (Ho & Liu, 2002; Bowe & Lee, 2004; Garvin & Cheah, 2004; Brandão & Saraiva, 2008; Ashuri et al., 2012; Liu et al., 2014; Carbonara & Pellegrino, 2018), as well as to analyse optimal price schemes (Feng et al., 2015). However, these are not the only compensation mechanisms, it is also usual to find compensation mechanisms for early termination (Caselli et al., 2009; Cabero et al., 2017; Liu et al., 2017; Igrejas et al., 2017; Song et al., 2018; Buso et al., 2019; Giraldo, 2019; Xiong & Han, 2021).

In that area, Caselli et al. (2009) priced the indemnification mechanism that ensures compensation for the private party when government terminates a BOT contract in the operation phase. Furthermore, Cabero et al. (2017) assessed the insolvency condition in motorway concessions in Spain where the public administration offers a mechanism of compensation to the concessionaire, with the value of the mechanism as an abandonment option depending on the evolution of traffic. Their results showed a significant value of the abandonment option, which represents an implicit aid from the public administration to the concession.

Liu et al. (2017) priced an early termination mechanism in PPP projects when the cash flow is excessively high and low by using the real options approach. They found that the termination mechanism is determined by the actual cash flow during the operation phase, the total investment and return on investment. Although, the total value of compensation agreed upon is also dependent on the bargaining power of each party. At the same time, Igrejas et al. (2017) implemented a real option model to value an embedded compound option to

abandon a PPP project. They focused on the value of the penalty to be imposed to discourage the project termination.

On the other hand, Song et al. (2018) analysed the early termination of PPP projects through an ex-ante compensation mechanism that considers the demand risk. They considered a win-win mechanism for both the public and private sector, through which the private sector gains a suitable compensation for the concession period. Similarly, Giraldo (2019) applied the ROT to value the early termination of a toll road project as a concessionaire's right to abandon the execution of the project, where the abandonment option depends on the risk of demand. In the model, the termination fee is determined by considering revenues, costs, and expenses as well as the investment value. By applying numerical methods, he found the termination fee acts as a revenue guarantee and as a guarantee to lenders in case of low traffic.

Furthermore, Buso et al. (2019) examined the effects of granting an exit option that enables the private party to terminate a PPP project early. They developed a model under a real options framework where the contractor holds private information on operating profits. Their results show that the exit option can soften agency problems and increase the government's payoff. Finally, Xiong & Han (2021) proposed a systematic approach by using the incomplete contract theory and the reference point theory to quantify the amount paid to compensate the private-sector partners efficiently when early termination occurs. They found this mechanism increases the benefits to the private sector but discourages its efforts to prevent early termination in serious risk scenarios.

Although the above work has made important contributions in this field, the value of the early termination mechanism requires a much more careful contractual design like Scott & Triantis (2004) and Guasch et al. (2007) suggested. Based on this work, we extend previous studies by incorporating optimal exercise policies to value early termination as an option to abandon.

2. Model Setup

2.1 Early Termination and the Compensation Mechanism

Usually, PPP projects consider compensation clauses to specify the parties' rights and obligations under early termination scenarios. According to Xiong et al. (2015), the compensation model is the core of the agreement. In that sense, the compensation model for early termination should be designed considering not only the potential risks but also the termination fee, which is determined

by several operational factors, one of them being the revenue of the project. According to Caselli et al. (2009), early termination could occur in two ways considering the decisions of each party involved in the contract:

- i) the government may consider the option to terminate the concession agreement if the infrastructure project becomes inadequate, or if the project only benefits the private party.
- ii) the private party may consider the option to terminate the concession agreement if the profits are not expected to be sufficient to guarantee a target return rate.

In both cases, private investors require a compensation fee paid by the government. For instance, in toll road infrastructure projects, investors have the right to receive fair compensation in case of loss of revenues due to unpredictable events, such as natural disasters or cost overruns. Such agreements promise an amount of compensation to investors under specified conditions.

Usually, the compensation fee is a stated fixed amount. For instance, the government can specify fixed levels of compensation considering the investment cost and expected profits for the operation phase. However, a fixed compensation amount for early termination is not adequate because the specific time where early termination will occur is unknown, and therefore, the termination fee should be determined by operational factors that change throughout the life of the project, like Giraldo (2019) suggested.

Even though there are no fixed rules to determine the best compensation mechanism, both the design and the valuation are relevant (Caselli et al., 2009). However, the concern of how to assign a valuation to the mechanism arises. The main concern is the contractual clause that gives private investors the right to receive a compensation fee when early termination occurs. In that sense, the real options theory (ROT) proposed in this paper arises as a useful tool to solve the problem (Cabero et al., 2017; Giraldo, 2019).

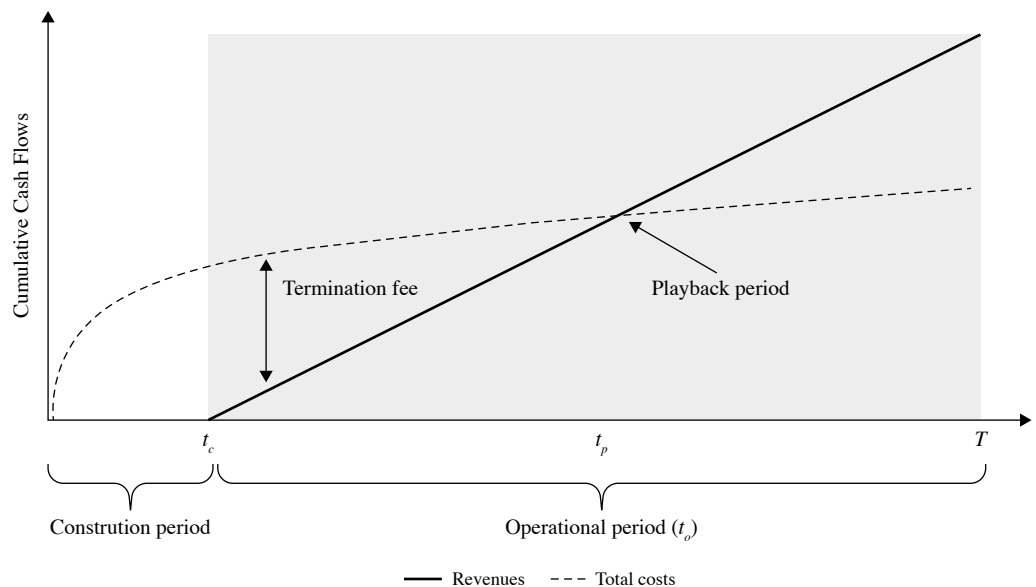
2.2 Early Termination under Real Options Theory

The decision of early termination of the concession (or abandonment) at any time t as an embedded option into the PPP agreement involves some flexibility about the uncertain future of the project. Therefore, the government must be able to predict the amount to be offered as the mechanism of compensation to

the private investors since that amount must be in the concession agreement. However, the exact value of the mechanism depends on the future of the project.

The compensation fee paid by the government could be determined as a deterministic formula based on any difference between the present value of costs (capex, opex, and maintenance, debt service and taxes) and revenues. By assuming that the construction period is t_c years, the operational period of the concessionaire is $T - t_c$ years, denoted by t_o , and the project is terminated at the T year, we determined the early termination fee until the investment of the project is recovered as shown in Figure 1.

Figure 1. Base projections of PPP project and early termination fee



Source: Adapted from Xiong et al. (2015).

In that case, we consider total costs as the amount of investment and operational costs until t_p , which is the payback period. Although real total costs could be either higher or lower than the base or estimated cost in PPP projects, we assumed that total costs are deterministic. Given that the revenues of the project depend on the real traffic levels and the toll rate, the government could compensate private investors by considering the difference between the present value of total costs and revenues. Because there is no downside protection in the project agreement, the analysis of traffic risk is crucial to assign a suitable

value to the compensation. The main concern is the estimation of revenues since revenues depend on a lot of risks. One of the most important risks is the traffic risk (or demand risk) like Brandão and Saraiva (2008) suggested, although other risks can cause significant losses for the concession and, therefore, these risks can lead to an early termination. For example, Zhang and Xiong (2015) analysed different types of risk events that often lead to early termination in PPP projects. Under this approach, the compensation fee is calculated as the present value (at time $t = 0$) taking the difference between the cumulated total cost minus the cumulated revenue at year t , as Xiong et al. (2015) suggested.

On the other hand, there is an important input to build the valuation model of the option to abandon based on the base termination fee. Financial theory has developed valuation models that can be considered as will be shown later. However, it should be taken into account that an analytical solution considering changes in the exercise price is more complex.

2.3 Analytical and Numerical Solutions

We fixed a complete probability space (Ω, F, P) with a filtration $(F_t)_{t \geq 0}$. Let $T_{t \geq 0}$ be an F_t – adapted traffic volume process. Under the filtered probability space, the traffic volume follows a geometric Brownian motion (gBm) process given by

$$dT_t / T_t = \mu dt + \sigma dW_t \quad (1)$$

where, μ and σ reflect the instantaneous drift and diffusion components respectively, and dW_t is an increment of a Wiener process under the real probability measure P . Additionally, like Brandão and Saraiva (2008), the observed revenue of the infrastructure in any year t , R_t , is given by:

$$R_t = T_t \times Toll\ rate \quad (2)$$

Where the *Toll rate* is assumed to be deterministic on lifetime. Operation and maintenance costs, including expenses and taxes, C_t , are deterministic throughout the lifetime of the project, and the infrastructure project generates a stochastic cash-flow process (X_t) , where X_t is a linear function of the R_t . Therefore, the present value of expected (future) cash flows denoted as V is given by

$$V = \int_0^T e^{-rt} E\{X_t | X_0 = x\} dt \quad (3)$$

To simplify the notation, we adopt the expression $V = (f(T))_{t \geq 0}$ where f denotes a known function. In fact, given that the X_t follows the stochastic process as $dX_t / X_t = \mu dt + \sigma dW_t$ by assuming the same parameters from T_t , and applying Ito's lemma, the diffusion process for V is obtained as

$$dV_t / V_t = \mu dt + \sigma dW_t \quad (4)$$

Now, the total investment cost (I) is assumed to be deterministic throughout the construction period (t_c): $I = \int_0^{t_c} e^{-rt} i_t dt$, where i_t is the annual investment cost required by the construction of the infrastructure project. Therefore, the difference between V and I determines the net present value of the project.

Finally, flexibility can be valued as an option to abandon and is equivalent to an American put option with an exercise price equal to the value of abandonment – this is the termination fee of the project. Nevertheless, like Giraldo (2019) suggested, the exercise of the option depends on the termination fee which is not fixed during the lifetime of the project. Therefore, valuation methods that incorporate this idiosyncratic feature of the early determination mechanism should be considered.

Letting $F = F(V)$ represent the value of the project considering the option to abandon in an early termination state if project conditions become adverse. By applying Ito's Lemma, we arrive at the following differential equation:

$$0.5\sigma^2 V^2 F''(V) + (r - \lambda)XF'(V) - rF(V) = 0 \quad (5)$$

where λ is the market risk premium using the capital asset pricing model (or CAPM). Now, let L be the fixed termination fee, then the value of the option to abandon can be determined as

$$F = \sup_{0 \leq \tau \leq T} E\{e^{-r\tau}(V_\tau - L_\tau)^+\} \quad (6)$$

Where $(V_\tau - L_\tau)^+$ is the payoff of the option to abandon and τ^* is the optimal stopping time. We propose adopting both analytical and numerical approaches to obtain a comparative framework about the solution from equation (6) to obtain the option value. In that sense, we used simple pricing schemes, for example

when the termination fee is fixed or when the termination fee changes over time, however this is also true for when the termination fee varies stochastically. The first model is the simplest pricing model.

Model 1: Termination fee is fixed

To obtain an analytical solution to price the option to abandon, we followed Clark & Rousseau (2002), although some theoretical insights of the model are provided in Myers and Majd (2001). The solution to equation (5) is

$$F = V + A_1 V^{\beta_1} + A_2 V^{\beta_2} \quad (7)$$

with $\beta_1 > 1$ because $r > 0$ and $\beta_2 < 1$. The roots to the quadratic equation $0.5\sigma^2 \beta^2 + (r - 0.5\mu^2) \beta - r = 0$ are:

$$\beta_1, \beta_2 = \left[-(r - 0.5\mu^2) \pm \sqrt{(r - 0.5\mu^2)^2 + 2\sigma^2 r} \right] / \sigma^2 \quad (8)$$

since the investment will not be abandoned as V gets larger and larger where $A_1 = 0$. Given the value of F depends then on V^* , the optimal abandonment value of the project is defined as a solution to the following boundary condition.

$$F(V^*) = L$$

Where the smooth pasting condition is given by

$$F'(V^*) = 0$$

And the general solution is

$$F = V + A_2 V^{\beta_2} \quad (9)$$

$$A_2 = -\frac{1}{\beta_2} V^{*1-\beta_2} \quad (10)$$

$$V^* = -\frac{\beta_2}{\beta_2 - 1} L \quad (11)$$

Thus, abandonment should take place when $V = V^* < L$ given that $\beta_2 < 0$.

Model 2: Termination fee is stochastic

Now, we consider the termination fee to vary stochastically according to a gBm process. Therefore,

$$dL_t / L_t = \alpha dt + \phi dZ_t \quad (12)$$

where α and ϕ reflect the instantaneous drift and diffusion components respectively, dZ_t is an increment of a Wiener process with $dW_t Z_t = \rho dt$ and ρ is the instantaneous correlation coefficient between V and L . In that sense, now F is a function:

$$F = F(V, L) \quad (13)$$

Following Clark & Rousseau (2002), we consider a new variable given by $g = V/S$ which represents the value of the investment per unit of termination value at time t . Applying Ito's lemma gives

$$dg = \lambda g dt = \gamma g ds \quad (14)$$

where:

$$\lambda = \mu - \alpha - \sigma\phi\rho + \phi^2 \quad (15a)$$

$$\gamma^2 = \sigma^2 - 2\sigma\phi\rho + \phi^2 \quad (15b)$$

$$ds = (\sigma dW - \phi dZ) / \gamma \quad (15c)$$

Letting

$$f(g, 1) = F(V, L) / L$$

and by applying Ito's lemma gives the following differential equation:

$$0.5\gamma^2 g^2 f''(g) + (r - \lambda) g f'(g) - r f(g) + r\lambda = 0 \quad (16)$$

Finally, similar to the boundary conditions in the previous model when the termination fee is fixed, the value matching condition is

$$f(g^*) = 1$$

and the smooth pasting condition is

$$f'(g^*) = 0$$

This gives the general solution:

$$f = g + K_1 g^{\eta_1} + K_2 g^{\eta_2} \quad (17)$$

where $\eta_1 > 1$ because $\lambda > 0$ and the roots to the quadratic equation are

$$\eta_1, \eta_2 = \left[-(r - \lambda - 0.5\gamma^2) \pm \sqrt{(r - \lambda - 0.5\gamma^2)^2 + 2\gamma^2 r} \right] / \gamma^2 \quad (18)$$

which is similar to the previous result. Therefore, the solution is:

$$f = g + K_2 g^{\eta_2} \quad (19)$$

where $\eta_2 < 0$, and

$$K_2 = -\frac{1}{\eta_2} g^{*1-\eta_2} \quad (20)$$

$$g^* = -\frac{\eta_2}{\eta_2 - 1} \quad (21)$$

Thus, abandonment should take place when $F = fL$. To compare the proposed model to price the option to abandon, we use numerical methods like the MC simulation technique where the option may be priced as an American style option following the work of Cabero et al. (2017), although the same results may be obtained by applying robust numerical models like the model proposed by Longstaff and Schwartz (2001). Additionally, in the comparative exercise, another valuation scheme may be adopted. In fact, Giraldo (2019) considered

a pricing formula to value the early termination fee where the termination fee depends on the revenues and the total costs of the project, $L = f(R, C)$, although both the revenues and the total costs are deterministic. Even though there are no analytical solutions to value the option to abandon in that sense, numerical methods arise as a practical solution to do that. Even though Giraldo (2019) used the Binomial model developed by Cox et al. (1979) to price the option, the MC simulation technique proves more useful, as Cabero et al. (2017) suggested. In the next section, we provide a numerical example where the valuation models discussed above are implemented.

3. Numerical Example

3.1 Assumptions and Deterministic Termination Fee

The proposed model is implemented using a hypothetical road infrastructure project to provide accurate estimations of both the present value of the project and the value of the option to abandon considering a fixed or a deterministic value of the termination fee as well as its stochastic dynamics. The project involves a toll road concession in Colombia under a BOT scheme. The project considers a road of 80 km that requires two years for the construction phase (starting in $t=1$) and eighteen years in the operation and maintenance phase. At the end of the period, the infrastructure will return to the public authority. The assumptions of the project are summarized in Table 1.

Table 1. Assumptions of the project

Project duration	20 yrs
Operational phase	18 yrs
Currency	USD
Annual inflation expected	3%
CAPEX (million) ⁱ	\$172
O&M costs (million)	\$3.0
Average toll rate	\$6.64
Average annual daily traffic (AADT)	7,200
Annual traffic growth:	4%

Tax rate:	32%
Equity	30%
MARR	10%
Debt ⁱⁱ	70%
Interest rate	6%
Loan term duration	10 yrs
Risk-free rate	3%

i. Total investment represents the resources needed by the infrastructure project and includes pre-operational costs, designs and financial costs for years 1 (55%) and 2 (45%).

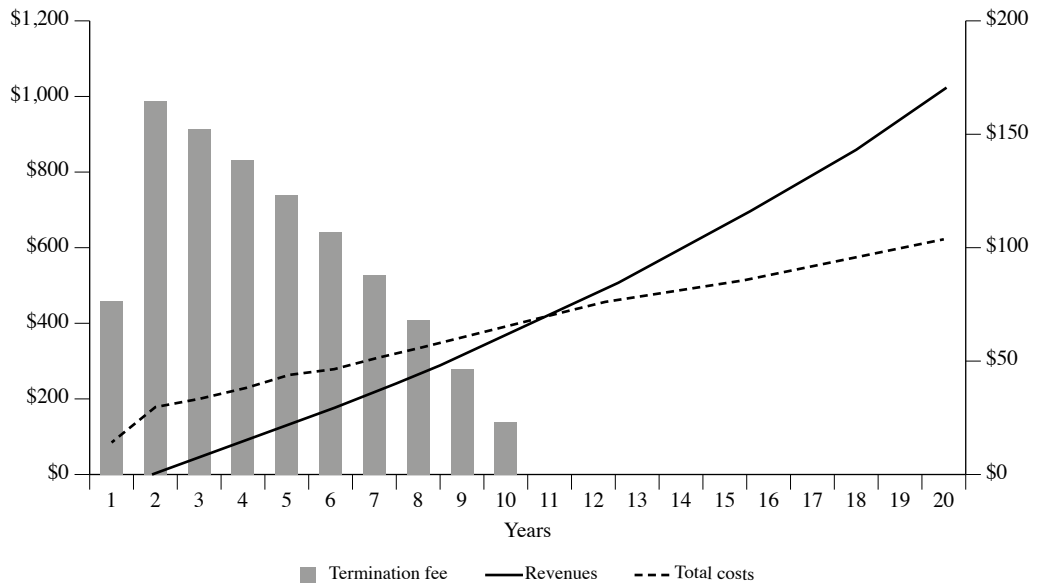
ii. It is assumed that the debt repayment scheme is fixed.

Source: Own elaboration.

Based on this information, the DCF analysis provides an NPV of \$200 million and an Internal Rate of Return (IRR) of 15.2%, by using a risk-adjusted discount rate ($WACC = 6,46\%$). Now, considering the forecasted cash flows from the project, we estimated the deterministic compensation fee (a base compensation line) associated with early termination of the concession by taking the present value from the difference between revenues and total costs based on the cash flow model, as Figure 2 shows. Additionally, it is assumed that there is not a penalty fee, and the government will compensate the total difference to private investors. For example, if the concession leaves the project at the end of the second year, once the construction phase ends, the government will compensate the value of \$164.28 million.

The total amount of public annual compensation (i.e., the termination fee) is estimated at the beginning of the construction phase until the eleventh (11) year on the operational phase for the concessionaire, which is the last year with a positive difference, and is the same time of the payback from the project. This means that private investors will recover their investment from this year onwards. Additionally, Figure 2 shows that the termination fee decreases from the third (3) year, given that the concession receives a toll revenue until the eleventh (11) year, and so that government will not assume any compensation fee in the remaining concession period.

Figure 2. Base compensation fee (deterministic model)



Source: Own elaboration.

Based on previous results, private investors have a base line to determine their decision to continue or abandon the concession. For instance, they could abandon the project at time t if the termination fee is greater than the PV of the difference (revenues-total costs). However, the estimation based on the cash flows model has notable limitations: not only does it omit the uncertainty sources of the project, but it also does not consider the stochastic nature of early termination of the concession and its flexibility. In that sense, it is necessary to complement the valuation analysis with the real options theory.

3.2 Pricing Early Termination Mechanism as a Real Option to Abandon

The decision to abandon the project is considered by incorporating the stochastic processes of the traffic volume process as well as the termination fee. In the valuation analysis, we consider different schemes of pricing to compare the value of the option to abandon. This comparative valuation exercise is carried out to show the limitations of adopting a fixed valuation scheme, as well as assuming a deterministic dynamic on the termination fee. To do that, we use analytical and numerical methods as we indicated before. The main

assumptions of the stochastic processes as well the parameters of the general solutions are summarized in Table 2.

Table 2. Parameters of the pricing model

V :	\$200
T :	11
r :	0.03
μ :	0.04
σ :	0.2
α :	0.04
ϕ :	0.16
ρ :	0.6

Source: Own elaboration.

Parameters such as V , T , μ and α were determined considering the results of the traditional DCF valuation model as well as the assumptions of the project showed in Table 1. The volatility of process for V (σ) was considered equal to the volatility of the traffic volume as we showed above. To do that, we estimated the value by considering historical data on traffic levels (20%). However, to determine the volatility of the process for L (ϕ) there was no data. As a result, we assumed that the parameter would be less than the volatility of V (16%). Finally, the correlation assumed ($\rho = 0,6$) means that relation between V and L is high, since L is determined by considering the difference between revenues and total costs provided by the same cash flow model. However, while there was no available data to determine a suitable correlation for this project, we complemented the valuation process with sensitivity analysis on this parameter and showed its effect on the option value. Based on the parameters of stochastic processes, we obtained the following results for the model proposed.

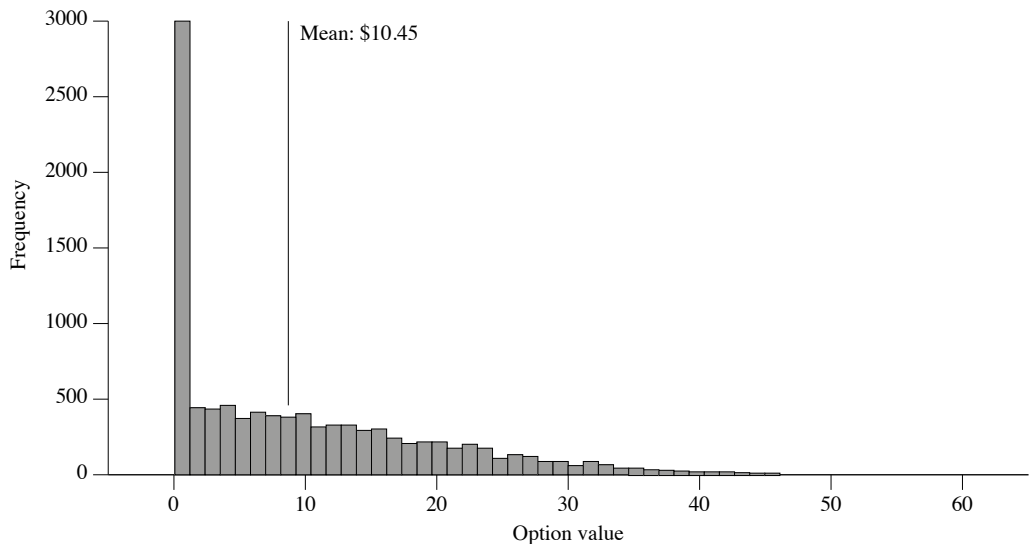
In the first scenario, we considered the simplest model to price the option to abandon. In this case, we priced the option to abandon according to equation (7)-(11) where the termination fee is equal to the present value of the total investment cost (\$151.88) million in $t = 0$. We found that the option to abandon with a fixed termination fee to be \$13.15 million, which means that early termination adds value to private investors. However, this option value

is accompanied by many limitations given that the termination fee cannot be fixed for the entire valuation period.

Now, following the work of Giraldo (2019), the termination fee is assumed to be deterministic during the first eleven (11) years of the project. Taking the estimated annual termination cost shown in Figure 1, we estimated the value of the option to abandon by applying the MC simulation technique like Cabero et al. (2017) suggested. To do so, the diffusion process for V is modelled according to equation (4) using the parameters shown in Table 2.

We performed 10,000 iterations of the stochastic process of V and, for each random trajectory, we estimated the option value by using equation (6). It is important to highlight that, in the valuation model, the exercise of the option and the optimal moment of exercise must be considered. Figure 3 shows the result for the option to abandon (\$10.45).

Figure 3. Option to abandon with a deterministic termination fee



Source: Own elaboration.

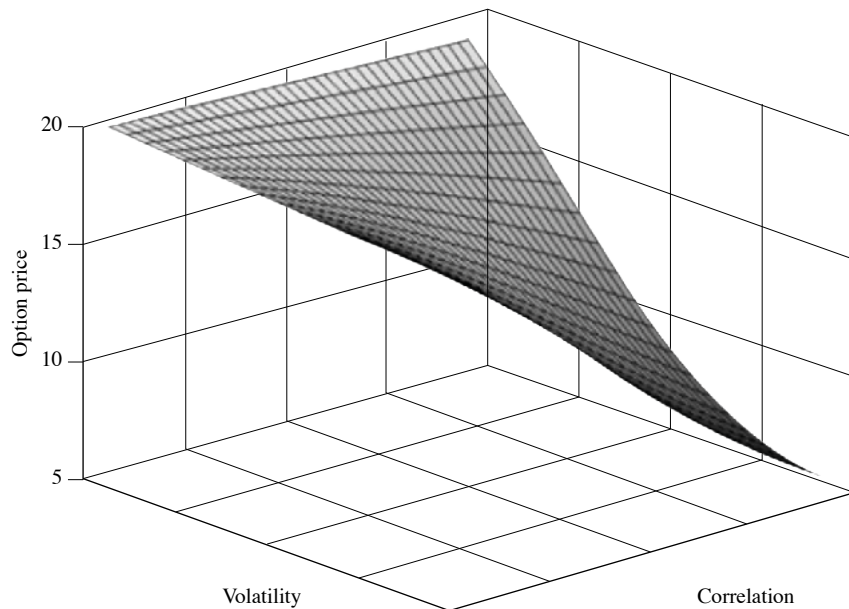
Finally, we implemented the proposed model to value the option to abandon when the termination fee is stochastic. In that case, we applied equations (14) and (17)-(21) where the parameters λ (0.0064) and γ^2 (0.1624) were calculated. The main results are summarized in Table 3.

Table 3. Option value with a stochastic termination fee

g^*	0,6732
$f(V, L)$	1,4
Option to abandon	\$12,46

The results suggest that the compensation mechanism adds \$12.46 million to private investors considering the stochastic processes for both V and L . However, it should be noted that this option value depends significantly on the assumed parameters. Therefore, we provide a sensitivity analysis to show how the option value responds to changes in the correlation (ρ) and volatility of L (ϕ) (See Figure 4).

Figure 4. Sensitivity analysis of the option value

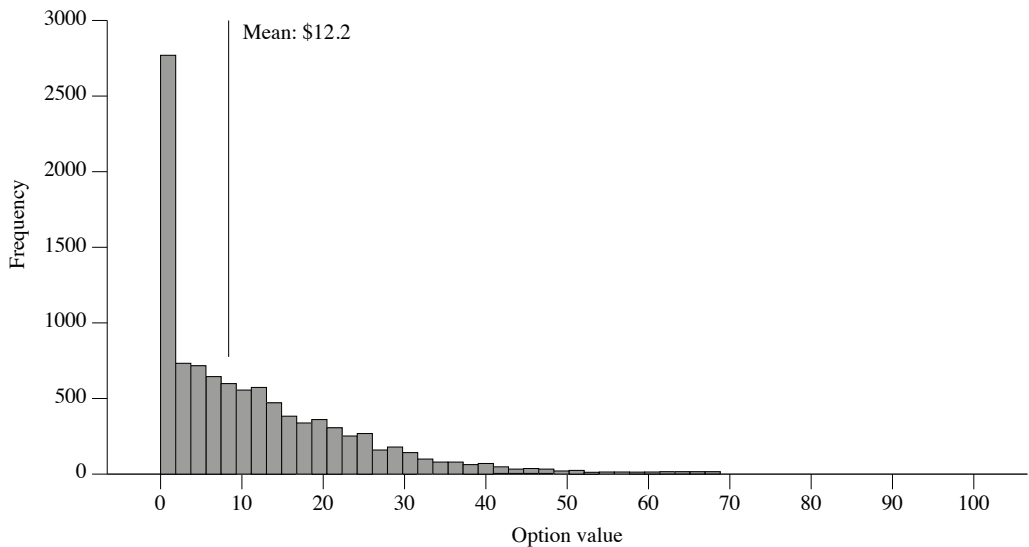


Source: Own elaboration.

To explain, the option value increases monotonically with the volatility and decreases monotonically with the value of the correlation. This means that the lower the correlation between V and L , the higher the option value, i.e., when the gBm processes are modelled independently, the compensation mechanism

provides a high value, but it can be mistaken. Hence the importance of taking a correlation that adequately reflects the relationship between V and L . Additionally, to compare the results with the MC simulation, we performed 10,000 iterations for the diffusion process of V and L . Therefore, the total present value of the option to abandon is obtained as shown in Figure 5. In this case, the value that the compensation mechanism provides to private investors is \$12.2 million.

Figure 5. Option to abandon with a stochastic termination fee



Source: Own elaboration.

4. Conclusions

In this paper we analysed a contractual compensation mechanism for the early termination of a hypothetical toll road infrastructure project. We focused on the early termination by the private investors, the option to abandon as an American put option was analysed and the valuation model based on real options theory was implemented. To do that, we considered stochastic processes to model both the traffic risk and the compensation fee. The option was then valued using analytical and numerical methods like the MC simulation technique. The proposed model shows a useful advantage compared to traditional models where idiosyncratic features from the compensation fee are incorporated in the

analysis. All this development shows the powerful advantages of the adopted model when compared to traditional models that consider a fixed or a deterministic termination fee.

Furthermore, the proposed model suggests that the mechanism for early termination could represent a strategy to mitigate the demand risk or the traffic risk for the private investors. What is more, it could be favourable to the government authority to promote infrastructure projects with private resources. However, there are still some limitations that could be overcome in future works. For example, models could be adopted for the optimal design of contracts, as well as the determination of the optimal moment in which to extend the contract to avoid the early termination of the concession.

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